

Cheap Money and Risk Taking: Opacity versus Fundamental Risk^{*†}

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Abstract

We explore the effect of interest rates on risk taking and find that the direction of the effect depends on the type of risk involved. In a Bayesian setting, investments can be risky either because they are opaque—i.e., their payoff-relevant signals are noisy—or because they are fundamentally risky—i.e., the dispersion of the prior is high. While both types of risk contribute symmetrically to the overall riskiness of an investment project, we show that changes in interest rates affect risk taking in these two types of risk very differently: when interest rates are high, investors choose transparent projects that are fundamentally risky; when interest rates are low, they choose opaque projects that are fundamentally safe. This makes the net effect of interest rates on risk taking necessarily ambiguous and dependent on the *sources* of risk.

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1 Introduction

Fueled by the recent financial crisis, a debate has been raging about the effect of interest rates on risk taking. Some observers, such as Taylor (2009a, 2009b), have argued that the crisis was a consequence low interest rates that led to excessive risk taking. Others, such as Johnson and Kwak (2010), have questioned the importance of low interest rates and, instead, have emphasized other potential causes for the crisis, such as a wholesale failure of regulation.

In this paper, we argue that an important element has been missing from this debate, namely, that the effect of interest rates on risk taking crucially depends on the *kind* of risk involved. In a Bayesian setting, the riskiness of an investment project is a combination of its “fundamental” risk and its “opacity” risk. Here, fundamental risk refers to the dispersion of the prior, while opacity risk refers to the noisiness of the payoff-relevant signal. While fundamental and opacity risk contribute symmetrically to the overall riskiness of an investment, we show that changes in interest rates affect risk taking in these two kinds of risk very differently: when interest rates are high, investors choose transparent projects that are fundamentally risky; when interest rates are low, they choose opaque projects that are fundamentally safe.

As a consequence, the net effect of a change in interest rates on risk taking is necessarily ambiguous. Indeed, we show that it can never be determined without knowing the *sources* of risk; i.e., the levels of opacity and fundamental risk of potential investment projects. When investment projects differ in terms of opacity but are relatively similar in terms of fundamental risk, low interest rates increase risk taking. If, on the other hand, potential investments differ in terms of fundamental risk but are relatively similar in terms of opacity risk, low interest rates decrease risk taking.

The simple intuition for these results is as follows. For high interest rates, only projects that are sufficiently “upgraded” in response to the observed signal receive funding. In this case, high volatility of posterior beliefs (reflected in conditional expectations that are a steep function of the signal) increases the chance that a project is financed. For low interest rates, by contrast, only projects that are sufficiently “downgraded” in response to their signal do *not* get funded. Now, volatile posterior beliefs *decrease* the chance that a project is financed. Finally, note that both greater transparency of signals and higher fundamental riskiness of payoffs induce more aggressive updating and, hence, raise the volatility of posterior beliefs about a project’s return. Opacity and fundamental safety, on the other hand, both reduce the volatility of beliefs. Therefore, high interest rates favor transparent projects with high fundamental risk, while low interest rates favor opaque projects with low fundamental risk.

Concretely, we study three variants of the same investment screening model. In all variants, investors know the type of project they are dealing with (i.e., opaque versus transparent and fundamentally safe versus fundamentally risky), while their funding costs are given by an exogenous real gross interest rate, R . In the first variant of the model, all investment projects have the same fundamental payoff risk but differ in their levels of opacity. That is, investors observe more informative signals about the future payoffs of “transparent” projects than about the payoffs of “opaque” projects. Here, we show that, above some threshold funding rate R^* , a fall in interest rates always increases the share of opaque investments in investors’ portfolios. This makes overall risk taking *decreasing* in the interest rate. When

the interest rate falls below R^* , there even exist strictly profitable transparent projects (i.e., projects with returns larger than R) that have a worse chance of financing than equally profitable opaque projects. Finally, we show that when expectations deteriorate during an economic downturn or crisis—modeled as a downward revision of prior beliefs—a “flight-to-transparency” occurs: the value of opaque projects is marked down more sharply than the value of transparent projects.

In the second variant of the model, all investment projects have the same level of opacity but differ in their fundamental riskiness. We show that all results derived in the first variant have an exact analogue in the second. However, the implications for risk taking are the opposite: in this case, a fall in interest rates raises the share of fundamentally safe projects in investors’ portfolios, which makes overall risk taking *increasing* in the interest rate.

Finally, we allow projects to differ both in terms of opacity and in terms of fundamental risk. This integrated model allows us to study the net effect of changes in interest rates on overall risk taking. We show that the net effect depends on the sources of risk and can never be determined from the overall riskiness of potential investment projects alone.

Empirically, it may be difficult to identify—let alone measure—opacity and fundamental risk. But the difference between the two can perhaps be illustrated in the case of structured financial products, such as collateralized debt obligations (CDOs). Arguably, senior tranches of CDOs combine high levels of opacity risk, due to their complex structuring, with low levels of (perceived) fundamental risk, due to tranching. According to our model, investments with these attributes should attract generous amounts of funding when interest rates are low. Indeed, this seems to be born out in the data: in 2006, after several years of low interest rates—particularly when judged against benchmarks such as the Taylor Rule—new CDO issuance reached a peak of \$1.1 trillion. Approximately 80% of the total were AAA-rated senior tranches (Baird, 2007). The model thus seems capable of explaining, at least in part, a potentially relevant stylized fact of the crisis. Still, our modeling approach remains incomplete in one important respect: by focusing on real investments and abstracting from financial markets, it cannot really predict how the two types of risk are priced and how they are allocated throughout the economy. Analyzing the interaction between secondary markets and the mechanisms governing the accumulation of the different risk types in reaction to changing levels of interest rates is a challenge that we leave for future research.

Related Literature In the operations research literature, Baker (2006) has studied the relationship between risk and informativeness. Even though her focus and set-up are different, the analyses are clearly related. Baker considers sequential decision problems and studies the effect on the first-period decision of a change in informativeness of the signal to be received before the second-period decision. By contrast, we consider one-shot investment decisions and study the effects on risk taking that stem from changes in the opportunity cost of investing.

Our paper is, of course, also closely related to the literature on the effect of interest rates on investor behavior and, in particular, risk taking. (See, e.g., Fishburn and Porter, 1976; Wong, 1997; Viaene and Zilcha, 1998; and, for a literature review, De Nicolò et al., 2010.) Note that in the classic Capital Asset Pricing Model, the market portfolio of risky assets becomes less risky when the risk-free interest rate declines—even if, ultimately, risk-averse

investors may choose to hold less of the riskless asset and more of the market portfolio. (See, e.g., Sharpe, 1964; and Lintner, 1965.) A fall in interest rates also decreases risk in basic models with asymmetric information, because lower interest rates reduce the adverse selection problem by making the loan applicant pool less risky (Stiglitz and Weiss, 1981).

In the wake of the recent financial crisis, the notion that low interest rates increase risk taking has become the more popular view. It often relies on the observation that low rates boost asset values. As asset values rise, balance sheets of banks grow, their leverage declines, and their risk taking and lending capacities expand. The greater willingness to lend may result in more risk taking to the extent that the set of safe borrowers is more or less fixed (Adrian and Shin, 2009). For non-banks, a similar dynamic is at play. Higher asset values increase collateral values and, thereby, create opportunities for additional risk taking on the part of investors. The motive for additional risk taking is then provided by the fact that risk tolerance tends to rise with wealth, such that higher asset values also make investors want to take on more risk. (Borio and Zhu, 2008; Gambacorta, 2009). Dell’Ariccia and Marquez (2006) demonstrate that low interest rates also reduce banks’ screening incentives and, thus, lead to riskier loan portfolios. Finally, Dell’Ariccia et al. (2010) suggest that the increase in bank leverage in response to low interest rates plays a key role in the link between monetary policy and risk taking.

In light of the extant literature, the contribution of the current paper is to clarify why the relationship between interest rates and risk taking is necessarily ambiguous: an investment typically involves both fundamental risk and opacity risk, while the propensity of investors to bear these two types of risk reacts in opposite ways to a change in interest rates.

2 Model

Consider a collection of investment projects, I . Each project $i \in I$ requires an investment of 1 today and produces a (real) payoff q next period. A project’s payoff is the realization of a random variable \tilde{q} , which is Lognormally distributed.¹ Specifically, $\tilde{q} = e^{\tilde{\pi}}$, where $\tilde{\pi}$ is Normally distributed with mean μ and variance σ_0^2 . Hence, the distribution of \tilde{q} is Lognormal with mean e^ν and standard deviation $e^\nu \sqrt{e^{\sigma_0^2} - 1}$, where $\nu \equiv \mu + \frac{1}{2}\sigma_0^2$. We call the variability of \tilde{q} the “fundamental” payoff risk of the investment project and measure it by the normalized payoff variance $\left(\frac{\text{Stdev}[\tilde{q}]}{E[\tilde{q}]}\right)^2 = e^{\sigma_0^2} - 1$ or, for convenience, simply by σ_0^2 .

A project’s payoff is not observable at the time of investment. However, before the investment decision is made, each project is costlessly screened by a potential investor; one for each project.² The screening produces a payoff-relevant signal, \tilde{y} , which is equal to the project’s π plus white noise. That is,

$$\tilde{y} = \pi + \tilde{\varepsilon}$$

¹This distributional assumption may seem rather restrictive. However, we will show that the basic intuition underlying our results carries over to all information systems and risk orderings that imply single-crossing of conditional expectation functions.

²If an investor screened multiple projects, his inference problem would become more complicated and dependent on the correlation between the payoffs and signals of different projects. To keep the problem manageable, we assume that projects are evaluated in isolation.

where $\tilde{\varepsilon}$, which is independent of $\tilde{\pi}$, is Normally distributed with mean zero and variance σ_1^2 . We call the noisiness of $\tilde{\varepsilon}$ the “opacity risk” of the project and measure it by the variance σ_1^2 . Signal \tilde{y} , whose CDF (PDF) we denote by F (f), is Normally distributed with mean μ and variance $\sigma_0^2 + \sigma_1^2$. The posterior distribution of \tilde{q} conditional on y is Lognormal with mean $E[\tilde{q} | y] = e^{(\sigma_1^2 \nu + \sigma_0^2 y) / (\sigma_0^2 + \sigma_1^2)}$ and standard deviation $E[\tilde{q} | y] \sqrt{e^{\sigma_0^2 \sigma_1^2 / (\sigma_0^2 + \sigma_1^2)} - 1}$.

We measure the “overall” riskiness of a project by the normalized payoff variance of its posterior, $\left(\frac{\text{Stddev}[\tilde{q}|y]}{E[\tilde{q}|y]}\right)^2 = e^{\sigma_0^2 \sigma_1^2 / (\sigma_0^2 + \sigma_1^2)} - 1$, or, for convenience, simply by $\sigma^2 \equiv \sigma_0^2 \sigma_1^2 / (\sigma_0^2 + \sigma_1^2)$. Notice that the two sources of risk—i.e., fundamental risk σ_0^2 and opacity risk σ_1^2 —contribute symmetrically to the overall riskiness, σ^2 , of a project.

Let r denote the real riskless interest rate in the economy, which constitutes the investors’ opportunity cost of investing. The gross cost of financing a project, which we denote by R , is then equal to $1 + r$. In the main text, we limit attention to equity financing by risk-neutral investors with sufficient funds of their own. In that case, a project is financed if and only if its expected return conditional on the signal, $E[\tilde{q} | y]$, is at least as large as the financing cost, R . In Appendix A, we show that all results remain unchanged when projects are fully or partially financed by debt (with limited liability for the equity investor). In Appendix B we show that, in a model with Normally—instead of Lognormally—distributed returns, we can accommodate risk aversion in the form of linear mean-variance utility. This does not change the basic intuition underlying our results.

In Section 3, we assume that all projects have the same fundamental risk, σ_0^2 , but differ in opacity risk, σ_1^2 . We analyze which projects are financed and how the share of opaque versus transparent projects in investors’ portfolios changes with the level of the interest rate. We also compare the average payoff of financed opaque projects with that of financed transparent projects and study relative valuation changes in response to changes in economic outlook or investor sentiment. In Section 4, we analyze the polar opposite case where all projects are equally opaque but differ in their levels of fundamental risk. In Section 5, we allow projects to differ both in terms of opacity and in terms of fundamental risk and focus on the net effect of changes in interest rates on overall risk taking. Finally, Section 6 concludes. Appendix C contains proofs relegated from the main text.

3 Transparent versus Opaque Projects

In this section, we consider the case where all projects in I have the same fundamental risk, but differ in their levels of opacity. Specifically, collection I contains two types of projects $\kappa \in \{P, T\}$, where P denotes opaque projects and T denotes transparent projects. Opaque projects have the same ex-ante expectation e^ν and fundamental risk σ_0^2 as transparent projects, but exhibit greater opacity σ_1^2 . That is, type P projects generate relatively uninformative signals \tilde{y}_P with variance $\sigma_{1,P}^2$, while type T projects generate relatively informative signals \tilde{y}_T with variance $\sigma_{1,T}^2 < \sigma_{1,P}^2$. Opaque projects make up a share $0 < \alpha < 1$ of projects in I , while the remaining projects are transparent. At the time of investment, investors know whether a project is of type P or type T .

A profit maximizing risk-neutral equity investor finances a project of type κ if and only

if the project's signal y_κ is such that $E[\tilde{q}_\kappa | y_\kappa] \geq R$. Solving for y_κ , we get

$$y_\kappa \geq \ln R + \frac{\sigma_{1,\kappa}^2}{\sigma_0^2} (\ln R - \nu) \equiv y_\kappa^R \quad (1)$$

Here, y_κ^R denotes the threshold signal such that $E[\tilde{q}_\kappa | y_\kappa] = R$.³

Financing Probabilities In this subsection, we investigate three related questions. First, we ask how the level of interest rates affects the relative financing probabilities of opaque versus transparent projects. Second, we study the effect of interest rates on the share of opaque versus transparent projects in investors' portfolios, i.e., among projects that are actually financed. Finally, we compare the average opacity and overall riskiness of financed projects with those of the project population at large.

Denote the probability that a random project of type κ is financed by p_κ^R . That is, $p_\kappa^R \equiv \Pr(\tilde{y}_\kappa \geq y_\kappa^R)$. The probability ratio p_P^R/p_T^R can be interpreted as the share of opaque projects in investors' portfolios, relative to their population share α . If this ratio is equal to 1, portfolio shares match population shares. Otherwise, one of the two types of projects is over-represented among financed projects. Define $\ln R^* \equiv \nu + \frac{1}{2}i_P i_T$, where $i_\kappa \equiv \sigma_0^2 / \sqrt{\sigma_0^2 + \sigma_{1,\kappa}^2}$. The following proposition describes how the interest rate affects the selection of investment projects.

Proposition 1 *If and only if $R \leq R^*$, then*

1. *Random opaque projects have a better chance of financing than random transparent projects.*
2. *Opaque projects are overrepresented in investors' portfolios.*
3. *Financed projects are, on average, more opaque and overall riskier than projects in I .*

Statements 1, 2, and 3 in the proposition are, in fact, equivalent. The intuition for the logical equivalence between low interest rates and statement 1 relies on the fact that payoff expectations are revised more strongly in response to transparent signals than in response to opaque signals. For high interest rates, only those projects receive funding that, in response to their signal, are sufficiently upgraded relative to prior beliefs. Because signals from opaque projects are less informative than signals from transparent projects, beliefs about opaque projects tend to be revised less than beliefs about transparent projects. This explains why random opaque projects are less likely to be funded than random transparent projects when interest rates are high. For low interest rates by contrast, in order to receive funding, projects must not disappoint too much. Hence, in this case, large (downward) belief revisions cause projects *not* to be funded. As before, opaque projects tend to experience smaller belief revisions than transparent projects. This explains why, for low interest rates, random opaque projects are *more* likely to be funded than random transparent projects.

³From (1) it is immediate that $y_P^R > y_T^R$ iff $\ln R > \nu$. However, from this observation we cannot conclude that an opaque project has a better chance of financing than a transparent project iff $\ln R > \nu$. Indeed, this is not true. The reason is that \tilde{y}_P and \tilde{y}_T are differently distributed and, hence, are not directly comparable.

To offer a graphical interpretation of Proposition 1 and better assess the robustness of the results, we now transform the signals \tilde{y}_κ through their CDFs into alternative but equivalent signals \tilde{z}_κ . That is, define

$$\tilde{z}_\kappa \equiv F_\kappa(\tilde{y}_\kappa)$$

By construction, the transformed signals \tilde{z}_P and \tilde{z}_T are identically distributed—namely, uniformly on $[0, 1]$. This makes them comparable in the following sense: in order to determine which type of project has a better chance of financing, it now suffices to check which of the transformed threshold signals, z_P^R or z_T^R , is smaller. Specifically,

$$p_P^R \geq p_T^R \iff z_P^R \leq z_T^R$$

Moreover, because signal distributions are uniform, the financing probabilities are simply $p_\kappa^R = 1 - z_\kappa^R$.

Figure 1 plots the conditional expectations $E[\tilde{q}_\kappa | z_\kappa]$, $\kappa \in \{P, T\}$, of opaque and transparent projects as a function of their normalized signals z_P and z_T . For each R , the cut-off z_κ^R is easily identified as the signal z_κ that makes $E[\tilde{q}_\kappa | z_\kappa] = R$. Note that the lower informativeness of \tilde{z}_P relative to \tilde{z}_T translates into $E[\tilde{q}_P | z_P]$ being “flatter” than $E[\tilde{q}_T | z_T]$. Hence, the former crosses the latter exactly once and from above at $R = R^*$, such that opaque projects have a better chance of financing than transparent projects if and only if $R \leq R^*$.

This graphical interpretation also clarifies that Proposition 1 is, in fact, quite robust and does not crucially depend on our specific distributional assumptions. Indeed, the result carries over to all payoff distributions and informativeness criteria that imply single-crossing of (normalized) conditional expectation functions. (See Ganuza and Penalva, 2010, for a discussion of informativeness criteria based on single-crossing of conditional expectations.)

Profitable Projects We now focus on the subset of profitable projects, i.e., those projects whose q_κ is strictly greater than the funding cost R . As profitable projects “have nothing to hide,” one might think that transparency would always increase profitable projects’ chances of financing. Consider, for example, the—admittedly extreme—case where transparent signals are perfectly informative. In that case, all profitable transparent projects, but only a fraction of profitable opaque projects, are funded. While this intuition may seem appealing, the next proposition shows that it is wrong.

Proposition 2 *If and only if interest rates are below the unconditional expected rate of return ($\ln R < \nu$), then there exist strictly profitable transparent projects that have a worse chance of financing than equally profitable opaque projects.*

To understand the intuition behind Proposition 2, consider a project that just breaks even. That is, the project’s actual payoff q_κ is equal to R , such that an investor would make neither a profit nor a loss. If signals are *almost* perfectly informative, then a break-even project has an approximately 50% chance of being financed, independent of R . If signals are almost uninformative, on the other hand, then the chances of financing do very much depend on R : when R is high relative to a random project’s unconditional expected payoff e^ν , then, for the usual reasons related to the sensitivity of the posterior with respect to the signal, a

highly opaque break-even project has almost no chance of financing. Hence, for high interest rates, a break-even project clearly stands a better chance when it is transparent than when it is opaque. When R is very low, the payoff expectation’s insensitivity to uninformative signals gives a break-even project almost guaranteed funding if it is opaque, while it still only has a 50/50 chance if it is highly transparent. Hence, now, a break-even project stands a better chance of being financed if it is opaque. Finally, by continuity, the same argument holds for strictly profitable projects with payoffs q_κ greater than, but close, to R .

Monotonicity We have seen that investors favor transparent projects when interest rates are high, and opaque projects when interest rates are low. We now show that, at interest rates above the unconditional expected rate of return, the shift from transparent to opaque projects is, in fact, monotone in R .

Proposition 3 *If interest rates exceed the unconditional expected rate of return, then a marginal reduction in interest rates raises the share of opaque projects in investors’ portfolios.*

Formally,

$$\ln R \geq \nu \implies \frac{d(p_P^R/p_T^R)}{dR} < 0$$

The monotonicity of the relative share of opaque projects, p_P^R/p_T^R , implies that a fall in interest rates increases the average opacity—and, thus, the average overall riskiness—of financed projects. Figure 2, which depicts p_P^R/p_T^R as a function of R , graphically illustrates both Propositions 1 and 3. Consistent with Proposition 1, the figure shows that for $R > R^*$, $p_P^R/p_T^R < 1$, such that opaque projects are underrepresented relative to their population share. Conversely, for $R < R^*$, opaque projects are overrepresented. Consistent with Proposition 3, the figure also shows that, for $\ln R \geq \nu$, the ratio p_P^R/p_T^R is a monotonically decreasing function of R . Therefore, a fall in the interest rate increases the share of opaque projects in investors’ portfolios. Even when $\ln R$ falls below ν (i.e., R falls below 1 in Figure 2), initially, the probability ratio p_P^R/p_T^R remains decreasing in R . Note, however, that when funding costs fall to zero, *all* projects in I are financed, irrespective of their type and signal. That is, $\lim_{R \downarrow 0} (p_P^R/p_T^R) = 1$. Therefore, by continuity, there must be a “turning point” smaller than R^* , below which the ratio p_P^R/p_T^R is upward sloping in R . In other words, the relative share of opaque projects cannot be globally decreasing in the interest rate.

Unlike Proposition 1, Proposition 3 does not carry over to all informativeness criteria that imply single-crossing of conditional expectations. However, the stronger assumption of supermodularity would suffice. (See Ganuza and Penalva, 2010, for a discussion of informativeness criteria based on supermodularity of conditional expectations.) Specifically, Proposition 3 holds for general information systems if the CDFs of the conditional expectations of opaque versus transparent projects are supermodular in the normalized signal, z . Supermodularity is a sufficient but not a necessary condition.

Average Payoffs of Financed Projects How does the average payoff of financed transparent projects compare with that of financed opaque projects? The answer to that question might seem obvious. Indeed, the fact that transparent and opaque signals can be ordered according to Blackwell’s (1953) sufficiency criterion implies that investors prefer

transparent signals over opaque ones. That is, investors’ ex-ante expected profits are increasing in the degree of signal transparency. However, from the fact that ex-ante expected profits are greater, one may not conclude that the average payoff of financed transparent projects is necessarily greater than that of financed opaque projects. For high interest rates, much fewer opaque than transparent project are undertaken. Hence, the average profit of financed opaque projects could very well be higher than that of financed transparent projects, while ex-ante average profits from a random opaque project remain lower.

Even though it is not implied by Blackwell (1953), the next Proposition shows that, for Lognormally distributed payoffs and Normally distributed signals, the average payoffs of financed transparent projects are indeed larger than those of financed opaque projects.

Proposition 4 *At any interest rate, the expected payoff of financed transparent projects is strictly greater than that of financed opaque projects.*

Change in Expectations During a financial crisis, investors are confronted with the fact that the prior beliefs on which their investment decisions were based were too optimistic. As a consequence, they adjust their priors downward. How does such a change in “sentiment” affect the market value of *existing* projects? Does it affect more strongly the value of opaque projects, or that of transparent ones?

We model a change in sentiment by a change μ . Note that a change in μ causes $\nu = \mu + \frac{1}{2}\sigma_0^2$ to change by the same amount.⁴ We say that investors become more pessimistic if μ falls and more optimistic if μ rises. The rate of change of the conditional expected return as a function of μ is

$$\gamma_\kappa = \frac{dE[\tilde{q}_\kappa | y_\kappa]}{d\mu} \frac{1}{E[\tilde{q}_\kappa | y_\kappa]} = \frac{\sigma_{1,\kappa}^2}{\sigma_0^2 + \sigma_{1,\kappa}^2}$$

Hence,

Proposition 5 *The value of opaque projects is uniformly more sensitive to changes in investor sentiment, as measured by μ , than the value of transparent projects.*

Formally, for all y_P and y_T , $\gamma_P > \gamma_T$.

According to Proposition 5, when expectations deteriorate during an economic downturn or crisis, a “flight-to-transparency” occurs: the value of opaque projects is marked down more sharply than the value of transparent projects. This may explain why, during the crisis of 2008, opaque financial instruments suffered dramatic declines in value—even those that were perceived to be relatively low-risk, such as senior tranches of CDOs.

The intuition for Proposition 5 is straightforward. The expectation of the posterior, $E[\tilde{q}_\kappa | y_\kappa]$, is (a monotone function of) a convex combination of the ex-ante expected value $\nu = \mu + \frac{1}{2}\sigma_0^2$ and the signal y_κ . The less informative the signal, the more weight is put on ν and the less on y_κ . Hence, opaque projects are more affected by revisions of μ than transparent projects.

⁴Of course, a change of ν can also come about through a change of σ_0^2 . However, if $y < \nu$, σ_0^2 has an ambiguous effect on a project’s conditional expected return. Therefore, we limit attention to changes in ν caused by changes in μ .

4 High versus Low Fundamental Risk

We now consider the case where all projects are equally opaque, but differ in their fundamental riskiness. That is, collection I contains two types of projects $\lambda \in \{H, L\}$, where H denotes fundamentally risky projects and L denotes fundamentally safe projects. Fundamentally risky projects have the same opacity, σ_1^2 , as fundamentally safe projects, but the variance of their priors is higher, i.e., $\sigma_{0,H}^2 > \sigma_{0,L}^2$. To ensure that both types of projects are equally attractive in ex-ante terms, we assume that (μ_H, μ_L) satisfy

$$\mu_H + \frac{1}{2}\sigma_{0,H}^2 = \mu_L + \frac{1}{2}\sigma_{0,L}^2 \equiv \nu$$

such that risky and safe projects have the same unconditional expected payoff, e^ν . Fundamentally risky projects make up a share $0 < \beta < 1$ of projects in I . The remaining projects are fundamentally safe.

While the analysis in this section is parallel to that in Section 3, the conclusions are the reverse. In Section 3 we saw that, under low interest rates, investors favor opaque—and, thus, overall risky—projects over transparent—and, thus, overall safe—projects. We now show that, under low interest rates, investors favor fundamentally safe—and, thus, overall safe—projects over fundamentally risky—and, thus, overall risky—projects. As we discuss in some detail in Section 5, these opposing effects make the net effect of changes in interest rates on overall risk taking highly ambiguous.

A risk-neutral equity investor finances a project of type λ if and only if the project's signal y_λ is such that $E[\tilde{q}_\lambda | y_\lambda] \geq R$. Solving for y_λ , we get

$$y_\lambda \geq \ln R + \frac{\sigma_1^2}{\sigma_{0,\lambda}^2} (\ln R - \nu) \equiv y_\lambda^R$$

Financing Probabilities Let $\ln R^{**} \equiv \nu + \frac{1}{2}i_H i_L$, where $i_\lambda \equiv \sigma_{0,\lambda}^2 / \sqrt{\sigma_{0,\lambda}^2 + \sigma_1^2}$. Once again, we begin by deriving an equivalence relation between the level of interest rates and, in this case, (i) the relative financing probabilities of fundamentally risky projects, (ii) the share of fundamentally risky projects in investors' portfolios, and (iii) the average fundamental riskiness and overall riskiness of financed projects.

Proposition 6 *If and only if $R \geq R^{**}$, then*

1. *Random fundamentally risky projects have a better chance of financing than random fundamentally safe projects.*
2. *Fundamentally risky projects are overrepresented in investors' portfolios.*
3. *Financed projects are, on average, fundamentally riskier and overall riskier than projects in I .*

Statements 1, 2, and 3 in the proposition are again equivalent. The logical equivalence between high interest rates and statement 1 is a consequence of single-crossing of the conditional expectation functions $E[\tilde{q}_H | z_H]$ and $E[\tilde{q}_L | z_L]$ (Figure 3). In Section 3, the flatness

of the conditional expectation of opaque projects relative to that of transparent projects implied that, at low interest rates, overall riskier projects stood a better chance of financing than overall safer projects. Conversely, at high interest rates, overall safer projects stood a better chance. In the current environment, these rankings are reversed. The flatness of the conditional expectation of fundamentally safe projects implies that, now, it is the overall safer projects that have a better chance of financing at low interest rates, while, for high interest rates, the financing probabilities of overall riskier projects are greater.

Projects with Identical Payoffs In Section 3, we saw that low-payoff projects have a better chance of financing if they are opaque rather than transparent, while high-payoff projects have a better chance of financing if they are transparent rather than opaque. For sufficiently low interest rates, we showed that this implies that some strictly profitable transparent projects are less likely to be financed than equally profitable opaque projects (Proposition 2).

For fundamentally risky versus fundamentally safe projects with the same payoff q , the ranking of financing probabilities turns out to be independent of q . Hence, if the interest rate is such that, for some q , a fundamentally risky projects has a greater (respectively, smaller) chance of financing than a fundamentally safe project with the same payoff, then this ordering holds for all q . Specifically,

Proposition 7 *Fundamentally risky projects with payoff q are more likely to be financed than fundamentally safe projects with the same payoff if and only if the interest rate exceeds the unconditional expected rate of return.*

Because the ranking of the financing probabilities in Proposition 7 holds for all payoffs q , it trivially also holds for projects with $q > R$, i.e., strictly profitable projects. Therefore, at high interest rates, all profitable fundamentally risky projects are more likely to be financed than equally profitable fundamentally safe projects, while at low interest rates the opposite holds.

Monotonicity The next proposition shows that the shift from fundamentally risky to fundamentally safe projects in response to changes in the interest rate is, once again, essentially monotone.

Proposition 8 *If the interest rate exceeds the unconditional expected rate of return, then a marginal reduction in the interest rate lowers the share of fundamentally risky projects in investors' portfolios.*

Formally,

$$\ln R \geq \nu \implies \frac{d(p_H^R/p_L^R)}{dR} > 0$$

Proposition 8 implies that, for $\ln R \geq \nu$, a fall in interest rates reduces the average fundamental riskiness (and, hence, overall riskiness) of financed projects. The proposition is illustrated in Figure 4, which depicts the relative share of financed fundamentally risky projects, p_H^R/p_L^R , as a function of the interest rate R . A sufficient condition for Proposition 8 to hold for general fundamental risk orderings and information systems is that the CDFs of the conditional expectation functions are supermodular.

Average Payoffs of Financed Projects Consistent with the pattern of reversal uncovered so far, we now show that financed fundamentally risky projects have higher average payoffs than financed fundamentally safe projects. The intuition is that fundamentally riskier projects have fatter right tails. This raises their expected payoff conditional on exceeding the threshold signal. In other words, the greater upside dominates the greater downside, which is mitigated by the screening procedure. Formally,

Proposition 9 *At any interest rate, the expected payoff of financed fundamentally risky projects is strictly greater than that of financed fundamentally safe projects.*

Change in Expectations Finally, how does a change in investor sentiment, μ , affect the market value of fundamentally safe projects relative to that of fundamentally risky projects? Because the rate of change as a function of μ of the conditional expected payoff $E[\tilde{q}_\lambda | y_\lambda]$ is $\gamma_\lambda \equiv \frac{dE[\tilde{q}_\lambda | y_\lambda]}{d\mu} \frac{1}{E[\tilde{q}_\lambda | y_\lambda]} = \frac{\sigma_1^2}{\sigma_{0,\lambda}^2 + \sigma_1^2}$, we obtain

Proposition 10 *The value of fundamentally safe projects is uniformly more sensitive to changes in investor sentiment, as measured by μ , than the value of fundamentally risky projects.*

Formally, for all y_H and y_L , $\gamma_H < \gamma_L$.

The intuition is again simple. When calculating $E[\tilde{q}_\lambda | y_\lambda]$, the safer the project, the more weight is put on the unconditional expectation $\nu = \mu + \frac{1}{2}\sigma_0^2$ and the less on the signal y_λ . Hence, the value of fundamentally safe projects is more sensitive to revisions in μ than the value of fundamentally risky projects.

5 Integrated Model

We now consider investment projects that differ both in terms of opacity and in terms of fundamental risk. This allows us to study the net effect of a change in interest rates on overall risk taking. Each project in I is characterized by a combination of prior and signal variances (σ_0^2, σ_1^2) . Projects are distributed on $[\underline{\sigma}_0^2, \bar{\sigma}_0^2] \times [\underline{\sigma}_1^2, \bar{\sigma}_1^2]$, where $0 < \underline{\sigma}_0^2 < \bar{\sigma}_0^2 < \infty$ and $0 < \underline{\sigma}_1^2 < \bar{\sigma}_1^2 < \infty$, according to some PDF $g(\sigma_0^2, \sigma_1^2)$. As before, all projects have the same unconditional expectation, e^ν , such that, in ex-ante terms, they are equally attractive to risk-neutral investors. This implies that $\mu = \nu - \frac{1}{2}\sigma_0^2$.

Iso-Risk, Iso-Probability, and Iso-Sensitivity Curves Recall that two projects are overall equally risky if and only if their posteriors have the same normalized conditional variance $\left(\frac{StDev[q|y]}{E[q|y]}\right)^2 = e^{\sigma^2} - 1$. Because this expression depends on (σ_0^2, σ_1^2) solely through $\sigma^2 = \sigma_0^2\sigma_1^2 / (\sigma_0^2 + \sigma_1^2)$, equally risky projects lie on “iso-risk” curves in the (σ_0^2, σ_1^2) -plane that are described by $\sigma_0^2\sigma_1^2 / (\sigma_0^2 + \sigma_1^2) = c$, where c is some positive constant (left panel of Figure 5).

Next, we identify classes of projects that have the same financing probabilities, p^R . Let $i \equiv \sigma_0^2 / \sqrt{\sigma_0^2 + \sigma_1^2}$ and note that i can be viewed as a measure of the contribution of fundamental risk to overall risk. Therefore, we refer to it as the “relative prior variance” of

the project. Recall that a project (σ_0^2, σ_1^2) is financed if and only if $E[\tilde{q}|z] \geq R$. This is equivalent to

$$z \geq \Phi \left(\frac{1}{i} (\ln R - \nu) + \frac{1}{2}i \right)$$

where Φ denotes the CDF of the standard Normal distribution. By construction, \tilde{z} is uniformly distributed. Hence, the financing probability, p^R , of project (σ_0^2, σ_1^2) is

$$p^R(\sigma_0^2, \sigma_1^2) = 1 - \Phi \left(\frac{1}{i} (\ln R - \nu) + \frac{1}{2}i \right)$$

The financing probability depends on (σ_0^2, σ_1^2) solely through the relative prior variance, i . Therefore, projects with identical relative prior variances have identical financing probabilities for all R and, thus, lie on the same “iso-probability curve.” Intuitively, the reason is that projects with the same i have the same conditional expectation functions, $E[\tilde{q}|z]$.

Because i is increasing in fundamental risk σ_0^2 and decreasing in opacity risk σ_1^2 , iso-probability curves are upward sloping in the (σ_0^2, σ_1^2) -plane. To see why, suppose that we raise fundamental risk and, thus, steepen the conditional expectation function $E[\tilde{q}|z]$. To get back onto the initial iso-probability curve we have to increase opacity, such that $E[\tilde{q}|z]$ becomes flatter again and takes on its original steepness. Therefore, iso-probability curves must be upward sloping.

Finally, we examine how a change in interest rates differentially *changes* the financing probabilities p^R of the various projects (σ_0^2, σ_1^2) . Because the derivative $dp^R(\sigma_0^2, \sigma_1^2)/dR$ also depends on (σ_0^2, σ_1^2) solely through the relative prior variance i , “iso-sensitivity curves” and iso-probability curves coincide (right panel of Figure 5).

Note that iso-risk curves are strictly downward sloping, while iso-sensitivity curves are strictly upward sloping. As a result, they intersect at most once. This implies that two projects with the same overall riskiness but different opacity and fundamental risk *never* exhibit the same interest rate sensitivity of financing probabilities. Conversely, projects with the same interest rate sensitivity but different opacity and fundamental risk *never* have the same overall riskiness. Therefore, we conclude

Remark 1 *The net effect of a change in interest rates on overall risk taking—as measured by the average riskiness of financed projects—can never be determined from merely knowing the overall riskiness, σ^2 , of projects. One also needs to know the sources of risk, namely, the levels of opacity and fundamental risk.*

Average Opacity and Fundamental Riskiness of Financed Projects In Section 3, we considered projects with different levels of opacity but the same fundamental risk and studied the effect of interest rates on the average opacity of financed projects. We showed that financed projects are on average more opaque—and, hence, overall riskier—than projects in I , if and only if interest rates are *low*. Then, in Section 4, we considered projects with different levels of fundamental risk but the same opacity and studied the effect of interest rates on the average fundamental risk of financed projects. There, we showed that financed projects are on average fundamentally riskier—and, hence, overall riskier—than projects in

I , if and only if interest rates are *high*. How do these results carry over to a setting where projects differ both in terms of opacity and in terms of fundamental risk?⁵

First, we derive a lemma that shows that, outside of an intermediate range of interest rates, a change in a project's relative prior variance, i , has an unambiguous effect on its financing probability. Let $\bar{i} \equiv \bar{\sigma}_0^2 / \sqrt{\bar{\sigma}_0^2 + \underline{\sigma}_1^2}$ and $\underline{i} \equiv \underline{\sigma}_0^2 / \sqrt{\underline{\sigma}_0^2 + \bar{\sigma}_1^2}$, and note that $0 < \underline{i} < i < \bar{i} < \bar{\sigma}_0$. Then,

Lemma 1

$$\begin{aligned} \frac{\partial p^R}{\partial i} < 0 & \text{ if } \ln R < \nu + \frac{1}{2}\underline{i}^2 \\ \frac{\partial p^R}{\partial i} > 0 & \text{ if } \ln R > \nu + \frac{1}{2}\bar{i}^2 \end{aligned}$$

The lemma is intuitive: Raising the relative prior variance makes the conditional expectation function steeper. For (sufficiently) high interest rates, this raises a project's financing probability. For (sufficiently) low interest rates, it reduces it.

The proof of Lemma 1 follows immediately from the fact that

$$\frac{\partial p^R}{\partial i} \begin{matrix} (>) \\ (<) \end{matrix} 0 \iff i \begin{matrix} (<) \\ (>) \end{matrix} \sqrt{2(\ln R - \nu)}$$

We use the lemma to show

Proposition 11 *Suppose $\tilde{\sigma}_0^2$ and $\tilde{\sigma}_1^2$ are independent. For sufficiently low interest rates (i.e., $\ln R < \nu + \frac{1}{2}\underline{i}^2$), financed projects are on average strictly more opaque and strictly fundamentally safer than projects in I . For sufficiently high interest rates (i.e., $\ln R > \nu + \frac{1}{2}\bar{i}^2$), financed projects are on average strictly more transparent and strictly fundamentally riskier than projects in I .*

Proposition 11 shows that, if fundamental risk and opacity risk are independent, then risk taking in these two types of risk responds to high and low interest rates as one would expect from the analyses in Sections 3 and 4. For (sufficiently) low rates, investors are drawn to projects with above-average opacity risk and below-average fundamental risk. For (sufficiently) high rates, the converse holds.⁶

It is worth noting that Proposition 11 does *not* hold for arbitrary distributions of $\tilde{\sigma}_0^2$ and $\tilde{\sigma}_1^2$. For example, suppose that all projects lie on the same iso-probability curve. In that case, all projects have the same financing probability and, hence, the average opacity and fundamental riskiness of financed projects is always the same as the average opacity and fundamental riskiness of projects in I . Thus, Proposition 11 fails.

⁵For ease of exposition, in Sections 3 and 4, we limited attention to binary project types—i.e., transparent versus opaque projects in Section 3, and fundamentally safe versus fundamentally risky projects in Section 4. Within the univariate settings of these sections, however, the monotone likelihood ratio properties derived in Propositions 3 and 8 allow the analyses to be readily extended to a continuum of types, without changing the results in an essential way.

⁶Independence of $\tilde{\sigma}_0^2$ and $\tilde{\sigma}_1^2$ is a sufficient but by no means necessary condition for the result in Proposition 11 to hold. In particular, the claim in the proposition will also hold for many (but not all) constellations where $\tilde{\sigma}_0^2$ and $\tilde{\sigma}_1^2$ are negatively correlated. Negative correlation does not guarantee the result, however, because the sign of the correlation between two random variables is not invariant under monotone transformations of these random variables. This means, for example, that even if $\tilde{\sigma}_0^2$ and $\tilde{\sigma}_1^2$ are negatively correlated and $\frac{\partial p^R}{\partial i} < 0$, then $\tilde{\sigma}_1^2$ and \tilde{p}^R are not necessarily positively correlated.

To complicate matters a bit, now suppose that all projects lie on a line in (σ_0^2, σ_1^2) -space that is uniformly steeper than the iso-probability curves. This means that an increase in fundamental risk, σ_0^2 , is associated with such a large rise in opacity, σ_1^2 , that the relative prior variance $i = \sigma_0^2 / \sqrt{\sigma_0^2 + \sigma_1^2}$ declines. In this example, let us compare the average fundamental risk of financed projects with the average fundamental risk of all projects. Due to the negative correlation between $\tilde{\sigma}_0^2$ and \tilde{i} , Lemma 1 implies that, for (sufficiently) low interest rates, projects with above-average fundamental risk σ_0^2 also have above-average financing probabilities. As a result, financed projects are on average fundamentally riskier than projects in I . Conversely, for (sufficiently) high interest rates, financed projects are on average fundamentally safer than projects in I . Hence, in a partial reversal of Proposition 11, for *low* interest rates, financed projects are on average *fundamentally riskier* than projects in I , while for high interest rates they are fundamentally safer.

Next, we compare the average opacity of financed projects with the average opacity of all projects. In this example, an increase in opacity σ_1^2 is associated with a sufficiently small increase in fundamental risk σ_0^2 such that no reversal of comparative statics occurs with respect to average opacity. That is, just as in Proposition 11, for *low* interest rates, financed projects are on average *more opaque* than projects in I , while for high interest rates they are more transparent. Combining these results we find that, for low interest rates, financed projects are on average both more opaque and fundamentally riskier than projects in I , while, for high interest rates, financed projects are on average both more transparent and fundamentally safer.

Analogous arguments show that if all projects lie on an upward-sloping line in (σ_0^2, σ_1^2) -space that is uniformly *flatter* than the iso-probability curves, then, for low interest rates, financed projects are on average both more transparent and fundamentally safer than projects in I , while, for high interest rates, financed projects are on average more opaque and fundamentally riskier.

From these examples, we can conclude that the net effect of low interest rates on risk taking can go either way. When projects lie on a line in (σ_0^2, σ_1^2) -space that is steeper than the iso-probability curves, then low interest rates raise the average opacity and fundamental riskiness of financed projects, such that overall risk taking increases. On the other hand, when projects lie on an upward-sloping line that is flatter than the iso-probability curves, then low interest rates reduce the average opacity and fundamental riskiness of financed projects, such that overall risk taking declines.

If fundamental risk and opacity risk are independently distributed, then opacity-risk taking rises with low interest rates, while fundamental-risk taking declines. The net effect on overall risk taking depends on the relative size of the increase in opacity-risk taking and the reduction in fundamental-risk taking, which, in turn, depends on the particulars of the distributions of opacity and fundamental risk.

Finally, irrespective of the correlation structure between the two types of risk, the following is true. When projects differ predominantly in terms of opacity rather than in terms of fundamental risk, then, essentially, the analysis in Section 3 applies: a drop in interest rates raises overall risk taking. If, on the other hand, projects differ predominantly in terms of fundamental risk rather than in terms of opacity, then, essentially, the analysis in Section 4 applies: a drop in interest rates lowers overall risk taking.

For the convenience of the reader, key results of the paper are summarized in Figure 6.

6 Conclusion

We have shown that the effect of interest rates on risk taking crucially depends on the kind of risk involved. At low interest rates, investors favor opaque investment projects that are perceived to be fundamentally safe. At high interest rates, investors are drawn to transparent projects that are fundamentally risky.

The fact that transparency and fundamental risk have the same effect on a project's chances of financing may seem paradoxical, since the former lowers risk, while the latter raises it. The paradox is resolved by observing that transparency and fundamental risk have the *same* effect on the volatility of investors' posterior beliefs, which determines the chance that a project is financed.

We have also shown that the net effect of a change in interest rates on risk taking can never be determined without knowing the *sources* of risk, i.e., the levels of opacity and fundamental risk of potential investment projects. If projects differ mostly in terms of opacity, then a drop in interest rates increases risk taking. If projects differ mostly in terms of fundamental risk, then a drop in interest rates decreases risk taking.

In our formal analysis, we have assumed that payoffs and signals are (Log)normally distributed. While this may seem restrictive, we have argued that the basic intuition underlying our results carries over to information systems and fundamental risk measures that imply single-crossing of conditional expectation functions. We have also assumed that the supply of investment projects and their risk characteristics are exogenously given. An interesting extension of the model would be to endogenize these aspects and study equilibrium pricing of opaque and fundamentally risky investments in financial markets with securitization. We leave this for future research.

A Debt Financing

Many complex financial instruments, such as mortgage-backed securities and collateralized debt obligations, are either debt instruments or some sort of hybrid between debt and equity. Here, we show that the results in the main text carry over to full or partial debt financing.

Suppose that a risk-neutral bank is asked to (co-)finance investment projects through a standard debt contract with limited liability for the equity investor. Which projects will be financed? First, consider the case where the bank is asked to finance projects in their entirety. In that case, at the margin, debt is effectively the same as equity. To see this, consider a marginal project with $\tilde{y}_\kappa = y_\kappa^R$, $\kappa \in \{P, T, H, L\}$. The bank will only be willing to fully finance such a project if the investor, who has no skin in the game, is willing to accept such a high interest rate, ρ , that the bank becomes the residual claimant and, hence, the *de facto* equity holder.⁷ Therefore, under full debt financing, a marginal project from the perspective of an equity investor is also marginal from the perspective of a debt investor, and vice versa. This means that exactly the same projects are undertaken regardless of whether they are financed through equity or debt contracts.

⁷For unbounded payoff distributions such as the Lognormal, $\rho \rightarrow \infty$. For bounded payoff distributions, the gross lending rate for marginal projects is equal to the upper bound of the payoff distribution.

What happens if financing is partly in debt and partly in equity? Suppose the bank finances a fraction δ , $\delta \in [0, 1]$, in the form of debt, while the investor puts up the remaining $1 - \delta$ in the form of equity. In a competitive lending market, the lending rate charged by the bank is determined by a zero-profit condition: it charges a gross lending rate ρ such that the expected total repayment to the bank is equal to δR —provided, of course, that such a $\rho < \infty$ exists. (If such a ρ does not exist, then the bank’s expected payoff is always less than δR in expectation and, hence, it is not willing to extend the loan.) For the equity investor to be willing to accept this lending rate, the residual payment he expects to receive, $E[\tilde{q}_\kappa | y_\kappa] - \delta R$, must be at least as large as his gross funding cost. That is,

$$E[\tilde{q}_\kappa | y_\kappa] - \delta R \geq (1 - \delta) R$$

which is equivalent to

$$E[\tilde{q}_\kappa | y_\kappa] \geq R$$

This condition is identical to the investor’s decision rule under equity financing. Hence, also for mixed debt-equity financing, the selection of projects is the same as under full equity financing. Note that the argument holds for all types of projects, i.e., opaque, transparent, fundamentally safe, and fundamentally risky. We summarize this observation in the next proposition.

Proposition 12 *The financing structure has no influence on which projects are undertaken. Hence, all propositions derived for equity financing carry over to the case of (full or partial) debt financing.*

B Risk Aversion

In the main text, we assumed that payoffs were Lognormally distributed and that investors were risk neutral. Here we show that, in a model with Normally distributed returns, we can accommodate risk aversion in the form of linear mean-variance utility, and that this leaves the basic intuition of our model unchanged.⁸

First, we claim that all results in the paper carry over, in essence, to the case of Normally distributed payoffs. While we do not give a formal proof, this claim is easily verified, also because the algebra for Normal payoffs is generally simpler than for Lognormal payoffs. (In the main paper, we nonetheless stuck to Lognormally distributed payoffs, in order to avoid negative returns and stay closer to the standard assumptions in the asset pricing literature.)

Next, let investor utility be given by

$$U(\tilde{q}) = E[\tilde{q}] - \zeta \cdot Var(\tilde{q})$$

where $\zeta \geq 0$. When $\tilde{q} \sim N(\mu, \sigma_0^2)$, a project’s utility conditional on a signal z is

$$\begin{aligned} U(\tilde{q}|y) &= E[\tilde{q}|z] - \zeta \cdot Var(\tilde{q}|z) \\ &= E[\tilde{q}|z] - \zeta \sigma_0^2 \sigma_1^2 / (\sigma_0^2 + \sigma_1^2) \end{aligned}$$

⁸With Lognormal payoffs, we can accommodate linear mean-standard deviation utility. However, unlike linear mean-variance utility, it is not obvious how to justify linear mean-standard deviation utility in an expected utility framework. (See, e.g., Samuelson, 1970.)

Hence, at gross interest rate R , a project is financed if and only if

$$E[\tilde{q}|z] \geq R + \zeta \sigma_0^2 \sigma_1^2 / (\sigma_0^2 + \sigma_1^2)$$

Thus, risk aversion makes the “effective” interest rate higher than the actual by $\zeta \sigma_0^2 \sigma_1^2 / (\sigma_0^2 + \sigma_1^2)$. Equivalently, it can be interpreted as shifting the conditional expectation function downward by the same amount. Because, for Normally distributed payoffs and signals, $E[\tilde{q}|z]$ is super-modular in z and σ_1^2 (respectively, σ_0^2), these shifts preserve single-crossing of the conditional expectations. Hence, the basic intuition underlying our main results carries over, even though the exact boundaries between “high” and “low” interest rates will be different.

For general risk preferences and information systems, the intuition continues to hold if and only if conditional expected utilities of the various types of projects satisfy single-crossing as a function of normalized signals z .

C Proofs

Proof of Proposition 1. The probability p_κ^R that a random project of type κ is financed is equal to

$$\begin{aligned} p_\kappa^R &= \Pr \left(y_\kappa \geq \ln R + \frac{\sigma_{1,\kappa}^2}{\sigma_0^2} (\ln R - \nu) \right) \\ &= 1 - \Phi \left(\frac{\ln R + \frac{\sigma_{1,\kappa}^2}{\sigma_0^2} (\ln R - \nu) - \mu}{\sqrt{\sigma_0^2 + \sigma_{1,\kappa}^2}} \right) \\ &= 1 - \Phi \left(\frac{1}{i_\kappa} (\ln R - \nu) + \frac{1}{2} i_\kappa \right) \end{aligned}$$

Therefore, $p_P^R \geq p_T^R$ iff

$$\ln R \leq \nu + \frac{1}{2} i_P i_T = R^*$$

This proves the equivalence between $R \leq R^*$ and statement 1.

The equivalence between statements 1 and 2 is trivial.

Finally, to prove the equivalence between statements 2 and 3, let $\hat{\sigma}_1^2$ denote the average opacity of financed projects, while $\dot{\sigma}_1^2$ denotes the average opacity of projects in I . Also, let s_P denote the share of opaque projects in investors’ portfolios. Then,

$$\begin{aligned} \hat{\sigma}_1^2 &= s_P \sigma_{1,P}^2 + (1 - s_P) \sigma_{1,T}^2 \\ &= \frac{\alpha p_P^R}{\alpha p_P^R + (1 - \alpha) p_T^R} \sigma_{1,P}^2 + \frac{(1 - \alpha) p_T^R}{\alpha p_P^R + (1 - \alpha) p_T^R} \sigma_{1,T}^2 \\ &= \frac{\alpha p_P^R / p_T^R}{\alpha p_P^R / p_T^R + (1 - \alpha)} \sigma_{1,P}^2 + \frac{(1 - \alpha)}{\alpha p_P^R / p_T^R + (1 - \alpha)} \sigma_{1,T}^2. \end{aligned}$$

Hence, $\hat{\sigma}_1^2 > \dot{\sigma}_1^2 = \alpha \sigma_{1,P}^2 + (1 - \alpha) \sigma_{1,T}^2$ iff $p_P^R / p_T^R > 1$. The comparison between overall riskiness of financed projects, $\hat{\sigma}^2$, and overall riskiness of projects in I , $\dot{\sigma}^2$, proceeds analogously.

■

Proof of Proposition 2. Let $p_\kappa^R(q)$ denote the probability that a project of type κ with payoff q is financed. Then,

$$\begin{aligned} p_\kappa^R(q) &= \Pr(y_\kappa \geq y_\kappa^R | q) = \Pr\left(y_\kappa \geq \ln R + \frac{\sigma_{1,\kappa}^2}{\sigma_0^2} (\ln R - \nu) \mid q\right) \\ &= 1 - \Phi\left(\frac{\ln R - \ln q + \frac{\sigma_{1,\kappa}^2}{\sigma_0^2} (\ln R - \nu)}{\sigma_{1,\kappa}}\right) \end{aligned}$$

Therefore, $p_T^R(q) < p_P^R(q)$ iff

$$\frac{\ln R - \ln q + \frac{\sigma_{1,P}^2}{\sigma_0^2} (\ln R - \nu)}{\sigma_{1,P}} < \frac{\ln R - \ln q + \frac{\sigma_{1,T}^2}{\sigma_0^2} (\ln R - \nu)}{\sigma_{1,T}}$$

which is equivalent to

$$\ln q < \ln R + \frac{\sigma_{1,P}\sigma_{1,T}}{\sigma_0^2} (\nu - \ln R)$$

Thus, if $\ln R \geq \nu$, then $p_T^R(q) < p_P^R(q)$ implies that $q < R$. Hence, the project is unprofitable. However, if $\ln R < \nu$, then all projects with $\ln q \in \left(\ln R, \ln R + \frac{\sigma_{1,P}\sigma_{1,T}}{\sigma_0^2} (\nu - \ln R)\right)$ are strictly profitable and satisfy the above inequality, such that they have a smaller chance of financing if they are transparent than if they are opaque. ■

Proof of Proposition 3. We prove the proposition by calculating $\frac{d}{dR} \left(\frac{p_T^R}{p_P^R}\right)$ and showing that it is positive for $\ln R \geq \nu$.

Let $x_T = \frac{1}{i_T} (\ln R - \nu) + \frac{1}{2}i_T$ and $x_P = \frac{1}{i_P} (\ln R - \nu) + \frac{1}{2}i_P$, and denote by Φ (respectively, ϕ) the CDF (PDF) of the standard Normal distribution. Then,

$$\frac{d}{dR} \left(\frac{p_T^R}{p_P^R}\right) = \frac{1}{R} \frac{\frac{1}{i_P} (1 - \Phi(x_T)) \phi(x_P) - \frac{1}{i_T} (1 - \Phi(x_P)) \phi(x_T)}{(1 - \Phi(x_P))^2}$$

This expression takes on the same sign as

$$\frac{i_T l(x_P)}{i_P l(x_T)} - 1 \tag{2}$$

where l is the hazard rate of the standard Normal distribution.⁹

If $\ln R$ is sufficiently large such that $x_P \geq x_T$ (which is equivalent to $R \geq R^*$), then the expression in (2) is clearly strictly positive and we are done. Hence, in the remainder, we assume that $x_P < x_T$.

⁹Recall that the hazard rate of the standard Normal distribution, $l(v)$, has the following properties:

1. $l(v)$ is strictly increasing and satisfies the differential equation $l'(v) = l(v)(l(v) - v)$;
2. $l(v) - v$ is strictly positive and strictly decreasing;
3. $l(v)/v$ is strictly decreasing for $v > 0$.

If $\ln R = \nu$, the expression in (2) simplifies to

$$\frac{i_T l\left(\frac{1}{2}i_P\right)}{i_P l\left(\frac{1}{2}i_T\right)} - 1$$

which is positive, because $\frac{l(v)}{l(w)} > \frac{v}{w}$ for all $0 < v < w$.

To establish the proposition, it now suffices to show that $\frac{d(l(x_P)/l(x_T))}{d \ln R} > 0$ for $\nu \leq \ln R \leq \ln R^*$. We have,

$$l^2(x_T) \frac{d(l(x_P)/l(x_T))}{d \ln R} = \frac{l(x_T) l'(x_P)}{i_P} - \frac{l(x_P) l'(x_T)}{i_T}$$

Using $l'(v) = l(v)(l(v) - v)$, the right-hand side of the last equality is equal to

$$\begin{aligned} & l(x_T) l(x_P) \left(\frac{l(x_P) - x_P}{i_P} - \frac{l(x_T) - x_T}{i_T} \right) \\ & > l(x_T) l(x_P) \left(\frac{l(x_P) - x_P}{i_T} - \frac{l(x_T) - x_T}{i_T} \right) > 0 \end{aligned}$$

where the last inequality follows from $x_P < x_T$ and the fact that $l(v) - v$ is strictly decreasing in v . ■

Proof of Proposition 4. The expected payoff of a financed project of type κ is $E[e^{\tilde{\pi}_\kappa} | \tilde{y}_\kappa \geq y_\kappa^R]$. Now,

$$\begin{aligned} E[e^{\tilde{\pi}_\kappa} | \tilde{y}_\kappa \geq y_\kappa^R] &= \int_{y_\kappa^R}^{\infty} E[e^{\tilde{\pi}_\kappa} | \tilde{y}_\kappa = y_\kappa^R] \frac{f(y)}{1 - F(y_\kappa^R)} dy \\ &= e^{\frac{\sigma_{1,\kappa}^2}{\sigma_0^2 + \sigma_{1,\kappa}^2} \nu} E \left[e^{\frac{\sigma_0^2}{\sigma_0^2 + \sigma_{1,\kappa}^2} \tilde{y}_\kappa} \mid \tilde{y}_\kappa \geq y_\kappa^R \right] \end{aligned}$$

Recall that if $\ln X \sim N(m, s^2)$, then $E[X | X \geq d] = e^{m + \frac{1}{2}s^2} \frac{1 - \Phi\left(\frac{\ln d - m - s^2}{s}\right)}{1 - \Phi\left(\frac{\ln d - m}{s}\right)}$. This implies that

$$E[e^{\tilde{\pi}_\kappa} | \tilde{y}_\kappa \geq y_\kappa^R] = e^\nu \frac{1 - \Phi\left(\frac{y_\kappa^R - \nu - \frac{1}{2}\sigma_0^2}{\sqrt{\sigma_0^2 + \sigma_{1,\kappa}^2}}\right)}{1 - \Phi\left(\frac{y_\kappa^R - \nu + \frac{1}{2}\sigma_0^2}{\sqrt{\sigma_0^2 + \sigma_{1,\kappa}^2}}\right)}.$$

Therefore, $E[e^{\tilde{\pi}_T} | \tilde{y}_T \geq y_T^R] > E[e^{\tilde{\pi}_P} | \tilde{y}_P \geq y_P^R]$ iff

$$\frac{1 - \Phi\left(\frac{y_T^R - \nu - \frac{1}{2}\sigma_0^2}{\sqrt{\sigma_0^2 + \sigma_{1,T}^2}}\right)}{1 - \Phi\left(\frac{y_T^R - \nu + \frac{1}{2}\sigma_0^2}{\sqrt{\sigma_0^2 + \sigma_{1,T}^2}}\right)} > \frac{1 - \Phi\left(\frac{y_P^R - \nu - \frac{1}{2}\sigma_0^2}{\sqrt{\sigma_0^2 + \sigma_{1,P}^2}}\right)}{1 - \Phi\left(\frac{y_P^R - \nu + \frac{1}{2}\sigma_0^2}{\sqrt{\sigma_0^2 + \sigma_{1,P}^2}}\right)}$$

Subbing $y_\kappa^R = \ln R + \frac{\sigma_{1,\kappa}^2}{\sigma_0^2} (\ln R - \nu)$ yields

$$\frac{1 - \Phi\left(\frac{1}{i_T} (\ln R - \nu) - \frac{1}{2}i_T\right)}{1 - \Phi\left(\frac{1}{i_T} (\ln R - \nu) + \frac{1}{2}i_T\right)} > \frac{1 - \Phi\left(\frac{1}{i_P} (\ln R - \nu) - \frac{1}{2}i_P\right)}{1 - \Phi\left(\frac{1}{i_P} (\ln R - \nu) + \frac{1}{2}i_P\right)}$$

And since $i_T > i_P$, this inequality follows from Lemma 2 below. ■

Lemma 2 *The function*

$$\theta(i) = \frac{1 - \Phi\left(\frac{1}{i} (\ln R - \nu) - \frac{1}{2}i\right)}{1 - \Phi\left(\frac{1}{i} (\ln R - \nu) + \frac{1}{2}i\right)}$$

is strictly increasing in i for $i > 0$.

Proof. Let $x = \frac{1}{i} (\ln R - \nu) + \frac{1}{2}i$ and $y = \frac{1}{i} (\ln R - \nu) - \frac{1}{2}i$, and note that $x = y + i$. Differentiating $\theta(i)$, we get

$$\begin{aligned} \theta'(i) (1 - \Phi(x))^2 &= \phi(y) \left(\frac{1}{i^2} (\ln R - \nu) + \frac{1}{2}\right) (1 - \Phi(x)) \\ &\quad - \phi(x) \left(\frac{1}{i^2} (\ln R - \nu) - \frac{1}{2}\right) (1 - \Phi(y)) \end{aligned}$$

which has the same sign as

$$\begin{aligned} &l(y) \left(\frac{1}{i^2} (\ln R - \nu) + \frac{1}{2}\right) - l(x) \left(\frac{1}{i^2} (\ln R - \nu) - \frac{1}{2}\right) \\ &= (l(y) - l(x)) \frac{1}{i^2} (\ln R - \nu) + \frac{1}{2} (l(y) + l(x)) \end{aligned} \quad (3)$$

where $l(\cdot)$ denotes the hazard rate of the standard Normal distribution.

Because $x > y$, this expression is clearly positive for $\ln R \leq \nu$. Hence, it remains to consider the case $\ln R > \nu$.

The expression in (3) can be written as

$$\begin{aligned} &-\frac{l(y+i) - l(y)}{i} \frac{1}{i} (\ln R - \nu) + \frac{1}{2} (l(y) + l(y+i)) \\ &> -\frac{1}{i} (\ln R - \nu) + \frac{1}{i} (\ln R - \nu) = 0 \end{aligned}$$

where the inequality follows from $\ln R > \nu$ and the fact that $l'(v) < 1$ and $l(v) > v$, for all $v \in \mathbb{R}$. ■

Proof of Proposition 6.

The proof is analogous to that of Proposition 1. ■

Proof of Proposition 7.

The probability $p_\lambda^R(q)$ that a project of type λ with payoff q will be financed is equal to

$$\begin{aligned} p_\lambda^R(q) &= \Pr(y_\lambda \geq y_\lambda^R | q) = \Pr\left(y_\lambda \geq \ln R + \frac{\sigma_1^2}{\sigma_{0,\lambda}^2} (\ln R - \nu) | q\right) \\ &= 1 - \Phi\left(\frac{\ln R - \ln q + \frac{\sigma_1^2}{\sigma_{0,\lambda}^2} (\ln R - \nu)}{\sigma_1}\right) \end{aligned}$$

Therefore, $p_L^R(q) < p_H^R(q)$ iff

$$\frac{\ln R - \ln q + \frac{\sigma_1^2}{\sigma_{0,H}^2} (\ln R - \nu)}{\sigma_1} < \frac{\ln R - \ln q + \frac{\sigma_1^2}{\sigma_{0,L}^2} (\ln R - \nu)}{\sigma_1}$$

which is equivalent to $\ln R > \nu$. ■

Proof of Proposition 8. The proof is analogous to that of Proposition 3. ■

Proof of Proposition 9. The proof is analogous to that of Proposition 4. ■

Proof of Proposition 11. Let $\hat{\sigma}_0^2$ and $\hat{\sigma}_1^2$ denote the average fundamental risk and opacity of financed projects, while $\dot{\sigma}_0^2$ and $\dot{\sigma}_1^2$ denote the average fundamental risk and opacity of projects in I . Financed projects are distributed on $[\underline{\sigma}_0^2, \bar{\sigma}_0^2] \times [\underline{\sigma}_1^2, \bar{\sigma}_1^2]$ according to the PDF $h(\sigma_0^2, \sigma_1^2)$.

From Lemma 1, we know that for $\ln R < \nu + \frac{1}{2}\dot{\nu}^2$, $\frac{dp^R}{d\dot{\nu}} < 0$ for all $(\sigma_0^2, \sigma_1^2) \in [\underline{\sigma}_0^2, \bar{\sigma}_0^2] \times [\underline{\sigma}_1^2, \bar{\sigma}_1^2]$. Hence, in that case, $p^R(\sigma_0^2, \sigma_1^2)$ is strictly increasing in σ_1^2 and strictly decreasing in σ_0^2 . Independence between $\tilde{\sigma}_0^2$ and $\tilde{\sigma}_1^2$ then implies that $\tilde{\sigma}_1^2$ and $\tilde{p}^R(\tilde{\sigma}_0^2, \tilde{\sigma}_1^2)$ are strictly positively correlated. Thus,

$$\begin{aligned} \hat{\sigma}_1^2 &= \int_{\underline{\sigma}_0^2}^{\bar{\sigma}_0^2} \int_{\underline{\sigma}_1^2}^{\bar{\sigma}_1^2} \sigma_1^2 \frac{p^R(\sigma_0^2, \sigma_1^2) g(\sigma_0^2, \sigma_1^2)}{\int_{\underline{\sigma}_0^2}^{\bar{\sigma}_0^2} \int_{\underline{\sigma}_1^2}^{\bar{\sigma}_1^2} p^R(\sigma_0^2, \sigma_1^2) g(\sigma_0^2, \sigma_1^2) d\sigma_1^2 d\sigma_0^2} d\sigma_1^2 d\sigma_0^2 \\ &= \frac{E[\tilde{\sigma}_1^2 \tilde{p}^R(\tilde{\sigma}_0^2, \tilde{\sigma}_1^2)]}{E[\tilde{p}^R(\tilde{\sigma}_0^2, \tilde{\sigma}_1^2)]} = \frac{\dot{\sigma}_1^2 E[\tilde{p}^R(\tilde{\sigma}_0^2, \tilde{\sigma}_1^2)] + Cov(\tilde{\sigma}_1^2, \tilde{p}^R(\tilde{\sigma}_0^2, \tilde{\sigma}_1^2))}{E[\tilde{p}^R(\tilde{\sigma}_0^2, \tilde{\sigma}_1^2)]} \\ &= \dot{\sigma}_1^2 + \frac{Cov(\tilde{\sigma}_1^2, \tilde{p}^R(\tilde{\sigma}_0^2, \tilde{\sigma}_1^2))}{E[\tilde{p}^R(\tilde{\sigma}_0^2, \tilde{\sigma}_1^2)]} > \dot{\sigma}_1^2 \end{aligned}$$

A similar argument establishes that $\hat{\sigma}_0^2 < \dot{\sigma}_0^2$.

The proof that for $\ln R > \nu + \frac{1}{2}\dot{\nu}^2$, $\hat{\sigma}_1^2 < \dot{\sigma}_1^2$ and $\hat{\sigma}_0^2 > \dot{\sigma}_0^2$ is analogous. ■

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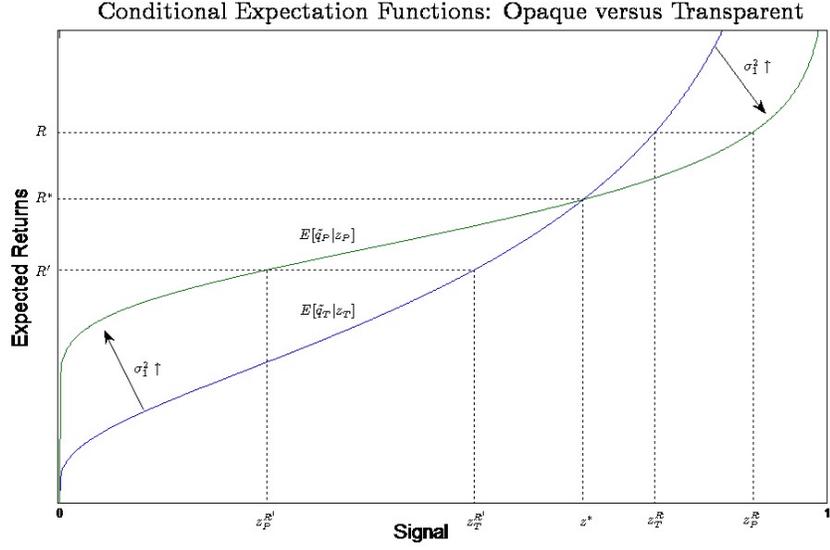


Figure 1: Conditional expectations of payoffs for transparent and opaque projects as a function of normalized signals z .

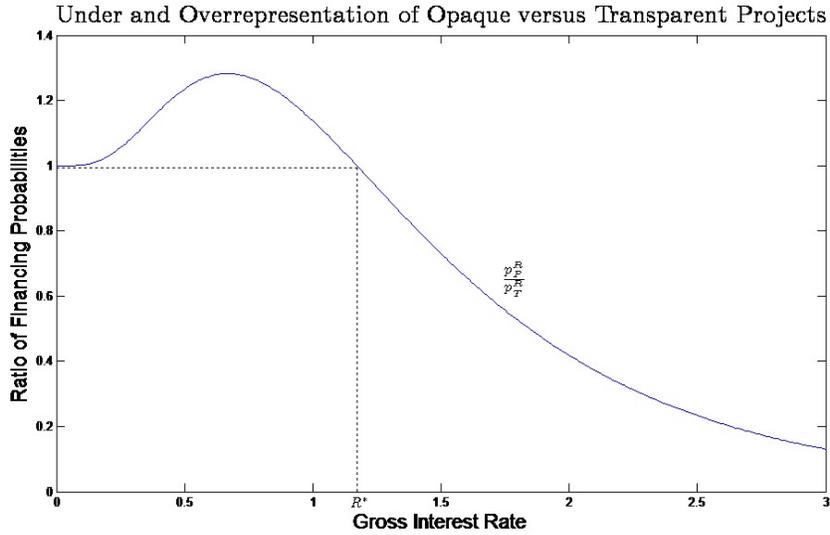


Figure 2: The ratio of financing probabilities as a function of the gross interest rate R . Opaque projects are overrepresented in investors' portfolios if and only if $R < R^*$. (Parameter values: $\sigma_0 = 1$, $\sigma_{1,T} = 1$, $\sigma_{1,P} = 2$, $\nu = 0$)

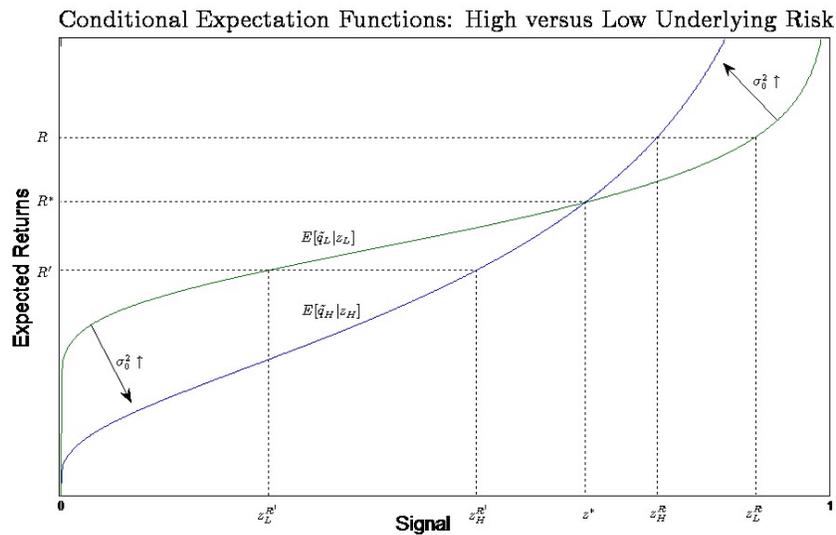


Figure 3: Conditional expectations of payoffs for fundamentally risky and fundamentally safe projects as a function of normalized signals z .

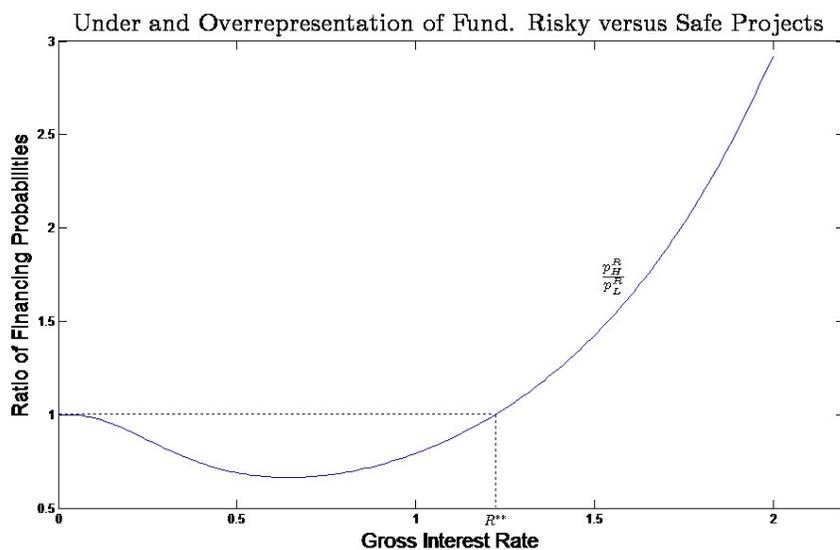


Figure 4: The ratio of financing probabilities as a function of the gross interest rate R . Fundamentally safe projects are overrepresented in investors' portfolios if and only if $R < R^{**}$. (Parameter values: $\sigma_1 = 2$, $\sigma_H = 1.5$, $\sigma_L = 1$, $\nu = 0$.)

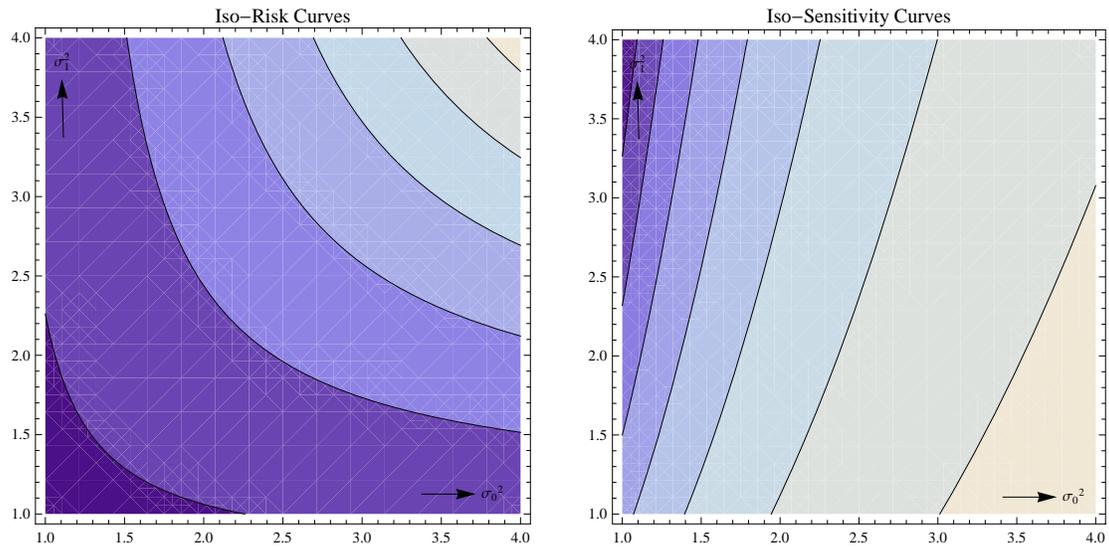


Figure 5: Iso-risk and iso-sensitivity curves. (Parameters: $\ln R = \nu = 0$.)

| TYPE OF RISK INTEREST RATE | SAME FUNDAMENTAL RISK, DIFFERENT OPACITY RISK | DIFFERENT FUNDAMENTAL RISK, SAME OPACITY RISK | INTEGRATED MODEL (Independent opacity and fundamental risk) |
|-------------------------------|---|--|--|
| HIGH | Transparent projects have better chance of financing. Financed projects are more transparent and overall safer. | Fundamentally risky projects have better chance of financing. Financed projects fundamentally riskier and overall riskier. | Financed projects are more transparent but fundamentally riskier . Ambiguous effect on overall risk. |
| LOW | Opaque projects have better chance of financing. Financed projects are more opaque and overall riskier. | Fundamentally safe projects have better chance of financing. Financed projects are fundamentally safer and overall safer. | Financed projects are more opaque but fundamentally safer . Ambiguous effect on overall risk. |
| AT ALL RATES | Value of opaque projects more sensitive to changes in sentiment than value of transparent projects. | Value of fundamentally safe projects more sensitive to change in sentiment than value of fundamentally risky projects. | |

Figure 6: Summary of key results