Estimating Potential Output for South Africa: A Production Function Approach

by

Harri Kemp

Abstract

Measuring potential output accurately is important. Judgements about the stance of the economy, medium term growth prospects and policy analysis depend on it. The aim of the paper is to review some of the methods employed in estimating potential output and to obtain a new estimate of potential output for the South African economy. Estimation of potential output is based on a structural production function. The reliability of the end sample estimates, the transparency and relative simplicity of the method, as well as the ability to examine the underlying determinants of potential output, provide the motivation for using the production function approach as the preferred methodology. A Cobb-Douglas production function is estimated for the period 1973 to 2010 and potential output calculated by substituting for potential levels of the factors in the estimated production function. Results indicate that a rigid labour market, characterised by consistently high levels of unemployment, a severe lack of skills, large numbers of discouraged and unemployable workers and strong labour unions, has restricted potential output growth. The estimates suggest that the potential sustainable growth rate (i.e. that level of output growth consistent with stable wage inflation) for South Africa is about 3.5%. This is substantially lower than the 7% target set by government. It appears as if the South African economy has hit capacity constraints, with potential growth far below official targets. Any policy aimed at increasing economic growth, while neglecting to address the various structural problems (specifically in the labour market) in the economy, will prove to be unsustainable.

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1. Introduction

Measuring potential output accurately is important. Judgements about the stance of the economy, medium term growth prospects and policy analysis depend on it.

Over the medium term, the growth rate in potential output provides a guide for the assessment of sustainable non-inflationary growth in output and employment. It is important for policy makers to know what the available (or excess) capacity of the economy is in order to design appropriate policies aimed at increasing the productive potential of the economy, with the goal of inducing sustainable long term growth.

The deviation of actual from potential output – the so-called output gap – provides a measure of relative supply and demand pressures in an economy at a particular time. The output gap therefore contains useful short-term information for the formulation of economic policy, specifically policies aimed at controlling inflation.

But potential output is hard to measure. Firstly, potential output is unobservable. Numerous concepts have been defined in the literature, but no consensus has been reached on the precise definition of potential output. Secondly, given the difficulty in pinning down a precise definition of potential output, a wide variety of methods are used in estimation exercises. Estimation results are often quite sensitive to the specific method employed and are in general not robust across methodologies.

The aim of the paper is to review some of the methods employed in estimating potential output and to obtain a new estimate of potential output (and hence the output gap) for the South African economy. Along the way the underlying factors driving changes in potential output are examined. The rest of the paper is organised as follows. Section 2 provides some background on the different methods used to estimate potential output, Section 3 outlines the methodology employed in this paper, Section 4 provides estimation results and Section 5 concludes.

2. Overview of Methods

The measurement of potential output is far from straightforward. Potential output can be derived from either a purely statistical approach or from full econometric analysis. The former attempts to separate a time series (such as real GDP) into trend and cyclical components using identifying assumptions based on statistical considerations, while the latter attempts to isolate the effects of structural versus cyclical influences on output using economic theory. Both have associated advantages and disadvantages, with the final choice of approach determined by the type of analysis and the purpose of the research.

Econometric analysis is appropriate for medium and long run studies, whereas the statistical approach may suffice for short run considerations. Statistical methods provide a mechanical way of obtaining estimates for potential output without resorting to economic arguments or the need to consider the underlying fundamentals driving its evolution. This approach would suffice over the short run as economic variables are subject to numerous, often transient, shocks which often result in a disconnect between the variable of interest and its underlying fundamentals.

Over the medium to long run, however, we expect to see a closer relationship between a specific variable and its underlying fundamentals. Therefore, in order to get a clear idea regarding the path of potential output, we need to investigate the fundamental driving forces behind its evolution. Additionally, it is important to obtain a clear understanding of the underlying fundamentals for the purposes of policy formulation and the investigation of the link between policy reform and actual outcomes.

2.1 Statistical Methods

The simplest statistical detrending technique is the linear time trend method[1]. The linear time trend method builds on the assumption that a series can be decomposed into a deterministic trend component and a cyclical component. It is assumed that the trend and cycle are fully uncorrelated (as opposed to the positive correlation found with the HP-filter, see McMorrow and Roeger, 2001). However, it is now generally accepted that the trend component is also subject to stochastic shocks. Consequently, if deterministic trends are used in order to calculate potential output and/or output gaps, there is a risk that stochastic trend elements may not be completely eliminated.

The Hodrick-Prescott (HP) filter (Hodrick and Prescott, 1997) attempts to overcome the above mentioned shortcoming of the linear time trend method and has become the most widely used detrending technique in the literature and at policy institutions[2]. As with the linear time trend method, the HP-filter assumes that time series' can be decomposed into a trend component and cyclical component and utilises a long run, symmetric, moving average technique to achieve the decomposition. The properties and shortcomings of the HP-filter are well documented (eg. Harvey and Jaeger, 1993; St Amant and Van Norden, 1997; Cerra and Saxena, 2000; McMorrow and Roeger, 2001; Mise, Kim and Newbold, 2003). Major drawbacks include the difficulty in identifying the appropriate detrending parameter, high end-sample bias (which reflects the two-sided moving average approach), the possibility of inducing spurious cyclicality when applying the filter to integrated or near-integrated series and excessive smoothing of structural breaks. On the other hand, the filter has some useful characteristics in that it allows for stochastic shocks to the trend component, it is relatively simple, transparent and requires minimal value judgement, is easily reproduced and is highly comparable across countries.

An extension of the basic HP-filter is the multivariate HP-filter. The multivariate HP-filter extends the HP-filter by adding at least one additional source of information (see Laxton and Tetlow, 1992; Butler, 1996) in an attempt to broaden the information set used to estimate unobserved potential output. The suggestion is that residuals from macroeconomic relationships that involve the output gap should be considered as additional conditioning variables in estimating trend output.

An alternative, albeit somewhat more complex, method to the above filtering techniques is the so-called Kalman filter/unobserved components approach (as pioneered by Harvey, 1992, amongst others). This technique assumes that macroeconomic time series are composed of distinguishable trend, cyclical and erratic/stochastic components. When interested in decomposing individual series (such as GDP) into trend and cycle components within a univariate framework, these components can be recovered from the observed series by imposing sufficient restrictions on the trend and cycle and implies assumptions about the functional form of these components and the structure of the error processes. A multivariate extension is also possible which allows for additional information to assist the decomposition process. The main attraction of the unobserved components approach lies in the fact that it offers a wide range of choices concerning the specification of the trend and cycle. It also allows for the possibility of using other empirical economic information for extracting cyclical

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[1] See Akinboade et al (2004) for an application to South Africa
[2] See Smit and Burrows (2002), Arora and Bhundia (2003) and Akinboade et al (2004), among others, for an application to South Africa

components. One disadvantage of the approach is that it requires considerable programming. Additionally, results are often sensitive to the initial choice of parameter values.

2.2 Econometric Methods

Econometric techniques for estimating potential output include the popular production function approach, the structural VAR (SVAR) approach and, more recently, the estimation of trend output utilising estimates obtained from dynamic stochastic general equilibrium (DSGE) models. The production function approach will be discussed in detail in following sections as this is the approach applied in this paper[3].

The general philosophy underlying the SVAR approach to estimating potential output rests on the theoretical idea that demand shocks are transitory, while supply shocks permanently affect output (i.e. affects output in the long run). The original Blanchard and Quah (1989) decomposition stems from the traditional Keynesian and neoclassical synthesis which identifies potential output with aggregate supply capacity and cyclical fluctuations with changes in aggregate demand. By imposing various long run restrictions on a system in output and other relevant variables (such as inflation and/or the real exchange rate), structural supply disturbances can be recovered after which the path of potential output can be derived using the resulting vector of supply shocks[4]. Blanchard and Quah suggest using economic theory to impose these identifying restrictions as it has several implications for the long-run behaviour of variables in response to shocks and therefore allows for the proper identification of the structural disturbances.

One of the advantages of the SVAR approach is that it relies on clear theoretical foundations and does not impose undue restrictions on the short-run dynamics of potential output. An obvious drawback is that the chosen identifying restrictions may not be applicable in all circumstances.

DSGE models have recently been utilised in an attempt to generate more robust estimates of potential output. In one such exercise, Julliard et al (2006) estimate a DSGE model for the US and use the forecasts of the model to generate robust end-of-sample estimates of the output gap in an attempt to improve on standard HP-filter results. However, measures of potential output obtained from DSGE models are dependent on the underlying characterisation of the economy employed in the specific modelling exercise. Changes in the underlying characterisation can lead to substantially different estimates of potential output.

2.3 Choice of Approach

The choice of approach is determined by the type of analysis and the purpose of the research. In this paper, the estimation of potential output is based on a structural production function. Cotis et al (2005) suggest that, when taking policy makers' needs into account, the production function approach is the preferred approach for estimating potential output. The use of this technique allows for the examination of the underlying economic factors driving changes in potential output, as well as the link between policy reform and actual outcomes.

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[3] See Smit and Burrows (2002), Arora and Bhundia (2003), Du Toit et al (2006) and Akinboade et al (2004) for an application of the production function approach; and Arora and Bhundia (2003) and Du Plessis et al (2007) for an application of the SVAR approach to South Africa

[4] The analysis can be extended in various ways. For example, one can include temporary nominal shocks by including a price variable affected by these nominal shocks in the short and long run (see Clarida and Gali, 1994)

It is however necessary that the "technical" performance of the approach against alternative methodologies should not be dramatically weaker across a range of criteria (e.g. internal consistency, transparency, information needed, ability to measure uncertainty in the estimates, end-sample reliability etc.). In this regard, the survey conducted by Cotis et al (2005) is reassuring when it comes to the feasibility of applying the production function approach in estimating potential output.

A particular strength of the production function approach is the reliability of the obtained estimates at the sample end points. As mentioned above, various other methodologies have the drawback of providing unreliable end sample estimates.

The reliability of the end sample estimates, the transparency and relative simplicity of the method, as well as the ability to examine the underlying determinants of potential output, provide the motivation for using the production function approach as the preferred methodology.

3. Estimating Potential Output

The particular concept of potential output modelled in this paper refers to the maximum level of output that is consistent with stable inflation. Therefore, the level of unemployment consistent with stable wage inflation is incorporated in the estimation.

Du Toit (1999) has shown that a Cobb-Douglas production function provides a useful estimate of the aggregate production function in South Africa. The Cobb-Douglas production function is represented by the following expression:

$$Y_t = A_t K_t^{\alpha} L_t^{\beta} \qquad (1)$$

with

$Y_t = actual\ gross\ domestic\ product$
$A_t = unobservable\ technological\ progress\ (including\ factor\ productivity)$
$K_t = actual\ capital\ stock$
$L_t = actual\ employment$
$\alpha = capital\ share\ parameter$
$\beta = labour\ share\ parameter$
$(\alpha + \beta)\ is\ restricted\ to\ 1 - assuming\ constant\ returns\ to\ scale$

Potential output is then obtained by substituting trend total factor productivity (A_t^*), actual capital stock (K_t)[5] and potential employment (L_t^*) back into the estimated production function.

3.1 Estimating Total Factor Productivity

The first step in estimating potential output is to obtain a measure of total factor productivity (A_t). The growth accounting framework, as pioneered by Solow (1957), is a popular method for obtaining estimates of total factor productivity. The method comprises factoring out the contributions of capital and labour inputs to output growth, where output is characterized by an aggregate production function (such as the Cobb-Douglas production function in (1)). Residual growth in output is then attributed to technological progress, or growth in total factor productivity. A problem with this approach is that

[5] The maximum potential output contribution of capital is given by the full utilisation of the existing capital stock in the economy

estimated total factor productivity includes omitted explanatory variables of growth and/or measurement error in the factors of production.

In this paper an alternative method for estimating total factor productivity is followed based on work by, amongst others, Fuentes and Morales (2006), Du Toit et al (2006) and Bonga-Bonga (2009). Given that total factor productivity is an unobserved variable, the latent variable or unobserved components approach seems a natural way to estimate it. To this end, the production function in (1) is recast in state-space form. This facilitates the simultaneous estimation of the elasticity parameters α and β ($= 1 - \alpha$) and total factor productivity (A_t), while allowing the latter to vary over time. The state-space representation of (1) is given as:

$$Y_t = \gamma_t K_t^{\alpha} L_t^{1-\alpha} e^{w_t} \qquad (2)$$

$$\gamma_t = \gamma_{t-1} + v_t \qquad (2')$$

with

$w_t, v_t = stochastic\ disturbances$
$\gamma_t = time\ varying\ paramter\ representing\ technological\ progress$

Equation (2) represents the so-called measurement equation and equation (2') represents the state (or transition) equation. The model is estimated using the Kalman-filter algorithm in Eviews[6]. Trend total factor productivity (A_t^*) is obtained by applying an HP-filter to the resulting series.

3.2 Estimating Potential Employment

Potential output is defined as the maximum level of output consistent with stable inflation. This implies that the actual labour input used in the estimated production function should be adjusted for the gap between actual unemployment and the estimated non-accelerating wage rate of unemployment (NAWRU). To that end, potential employment is defined as follows:

$$L^* = LFS(1 - NAWRU) \qquad (3)$$

with

$LFS = the\ smoothed\ labour\ force\ (product\ of\ labour\ force\ and\ the\ trend\ participation\ rate)$
$NAWRU = estimated\ non - accelerating\ wage\ rate\ of\ unemployment$

The method adopted to estimate the NAWRU (as in Elmeskov and MacFarlan, 1993; Pichelmann and Schuh, 1997; Du Toit et al, 2006) assumes that the rate of change in wage inflation is proportional to the gap between actual unemployment and the NAWRU. Assuming that the NAWRU changes only gradually over time[7], successive observations on wage inflation and actual unemployment rates can be used to calculate a series corresponding to the implicit value of the NAWRU:

$$D^2 \log W = -a(U - NAWRU), \quad a > 0$$

[6] The algorithm is discussed in detail in Appendix B
[7] This assumption is based on partial hysteresis: actual unemployment feeds only partly into long-run equilibrium unemployment; unemployment evolves only slowly towards its steady-state level

Where D is the difference operator, W is the real wage rate and U is the (actual) unemployment rate. Further assuming that the NAWRU is constant between any two consecutive periods, we have:

$$a = -(D^3 logW)/DU$$

This allows us to obtain an estimate of the short run NAWRU as follows:

$$NAWRU = U - \left(\frac{DU}{D^3 logW}\right) D^2 logW \qquad (4)$$

A popular alternative method for estimating the NAWRU (or NAIRU: non-accelerating inflation rate of unemployment) is to embed this unobserved variable in a standard Phillips-curve relation where inflation responds to the unemployment gap. The equation is recast in state-space (similar to the method followed above to obtain a measure of total factor productivity) and estimated using the Kalman filter. This framework allows incorporation of various other explanatory variables in the Phillips-curve relationship. More specifically, proxies for short run supply shocks can be included which results in a medium term estimate of the implicit NAIRU.

4. Estimation Results

Because some variables are only available at an annual frequency[8], estimation is carried out for the period 1970 to 2010 using annual data obtained from the South African Reserve Bank (SARB) and Quantec. A detailed description is supplied in the data appendix.

4.1 Total Factor Productivity

To facilitate estimation in Eviews, the system in (2) and (2') is recast in state space form as follows:

$$\log Y = C(1) * \log K + \big(1 - C(1)\big) * \log L + SV1 + \big[Var = \exp\big(C(2)\big)\big]$$

$$SV1 = SV1(-1) + \big[Var = \exp\big(C(3)\big)\big]$$

where $\log K$ and $\log L$ are the natural logarithms of capital and labour respectively. $SV1$ represents total factor productivity in the Cobb-Douglas specification and is time varying. $C(1)$ represents the capital share parameter (α). The exponential function of $C(2)$ represents the variance of the error term of the measurement equation and the exponential function of $C(3)$ represents the variance of the error term of the state equation[9]. The results of the estimation exercise are presented in Table 1. The capital share parameter, α, is estimated at 0.302 and corresponds well to earlier studies[10].

Insert Table 1 here

The resulting series for total factor productivity, along with the corresponding HP-filtered series, is presented in Figure 1.

[8] Using annual data precludes any analysis of the short-term cyclical properties of the resulting series for potential output and is one of the disadvantages of using the production function approach

[9] Using this specification, the variances of the innovations are restricted to be positive functions of the coefficients c(2) and c(3)

[10] The capital share parameter is at the lower end of the spectrum compared to previous estimates for South Africa, but makes intuitive sense given the relatively labour intensive nature of production

Insert Figure 1 here

It is clear that there has been a substantial pick-up in TFP growth in South Africa since the mid 1990's, with annual growth averaging 1.79% for 1994 to 2010 as opposed to -0.11% for 1971 to 1993. A growth accounting exercise based on the Cobb-Douglas production function estimated above suggests that the substantial increase in real GDP growth since the mid 1990's reflects the significant increase in TFP growth rather than factor accumulation. Table 2 shows the contributions to GDP growth of the different inputs across the two time periods.

Insert Table 2 here

The contributions of both capital and labour have decreased substantially, with TFP growth by far the largest contributor to real GDP growth since 1994. Recent studies on South African growth performance (Fedderke, 2002; Arora and Bhundia, 2003; Arora, 2005; Du Plessis and Smit, 2009) have also highlighted the turnaround in the contribution of total factor productivity.

In order to identify the factors driving TFP, one needs to provide some theoretical content to the concept of TFP growth. TFP growth may result from spill-overs associated with capital investment (Romer, 1986), spill-overs related to investment in human capital (Lucas, 1988), or from explicit investment in research and development, sometimes referred to as Schumpeterian TFP growth. Fedderke (2005) identifies the core determinants of South African TFP growth as a combination of factors determining the quality of human capital development and R&D activity. However, the results were based on data from the manufacturing sector alone (representing 20% of GDP). Generalising these results to the economy as a whole is suspect.

Arora and Bhundia (2003) suggest that the turnaround in TFP performance since 1994 can partly be attributed to the policy and institutional changes enacted during and after transition. The main argument is that international trade and investment opens the door for technological spill-over effects (i.e. Romer and Lucas-type spill-over effects), while greater private sector involvement in the economy increases the scope for technological innovation (Schumpeterian R&D effects). In South Africa, the scope for such effects increased dramatically with the increased openness of the economy and increased share of investment accounted for by the private sector after 1994. Indeed, Edwards (2005) and Edwards and Lawrence (2006) report evidence of liberalisation at the aggregate and sectoral level in South Africa since the late 1980's, while Jonsson and Subramanian (2001) attributes the bulk of TFP growth in South Africa during the 1990's to increased openness to trade.

Irrespective of the specific determinants driving TFP growth, the significance of the turnaround in the contribution of TFP to economic growth is that GDP growth can generally be sustained over longer periods when based on technological progress and improvements in efficiency rather than factor accumulation, which face natural limits based on demographics and diminishing returns (Arora and Bhundia, 2003).

4.2 The NAWRU

The estimated NAWRU is presented in Figure 2 along with the corresponding HP-filtered series. The estimated series compares favourably with the estimate in Du Toit et al (2006) in both the estimated level of the NAWRU and the general trend.

Insert Figure 2 here

Figure 2 shows that the South African NAWRU increased at a steady rate for much of the sample period, implying severe structural problems in the economy as a whole and the labour market in particular. These results imply that a unique equilibrium level of unemployment (i.e. the level of the NAWRU to which unemployment reverts in the long run) does not exist. The steadily increasing NAWRU can be attributed to the hysteresis nature of unemployment in South Africa. Pichelmann and Schuh (1997) present a theoretical model explaining the occurrence and effects of hysteresis on equilibrium unemployment while discussing the different mechanisms through which employment shocks may be perpetuated.

The hysteresis generating mechanism that is most applicable to the South African labour market operates through changes in human capital. According to this view, prolonged periods of unemployment may lead to the deterioration of skills, while also providing no opportunities for learning-by-doing and on-the-job training. Kingdon and Knight (2004) find that the average duration of uncompleted spells of unemployment in South Africa is 2.2 years, while Banerjee et al (2006) find that of adults aged 16 – 64 who are unemployed, only 9.6% find employment after six months. The long duration of unemployment spells for large sections of the population leads to substantial deterioration in skills and also leads to many workers becoming discouraged and reducing their job search intensity. The consequence is that the long term unemployed receive fewer and fewer job offers over time and may become unemployable: the mere fact of being unemployed for long periods of time may convey negative signals regarding productivity levels to prospective employers. The resulting disattachment from the labour market has the consequence that the long term unemployed exert little or no downward pressure on wages (Pichelmann and Schuh, 1997).

Another possible hysteresis mechanism that could lead to permanent shifts in equilibrium unemployment puts emphasis on the wage bargaining behaviour of employed insiders. The presence of many unionised workers may drive up real wages faster than productivity growth, thereby exacerbating unemployment (Banerjee et al, 2006). When unions bargain mainly on behalf of the incumbent workforce, a temporary shock to employment will tend to perpetuate itself as real wage demands are adapted to the now smaller number of employed insiders (Pichelmann and Schuh, 1997). This is especially true in South Africa where collective agreements reached by bargaining councils can be extended to all workers in all firms in an industry, even if they were not parties to the negotiations, while there is also scope for additional firm level negotiations (see Bendix, 2003). In general, shifts in the employment composition in favour of those facing little risk of unemployment may impact on the bargaining stance of unions and reduce the wage moderating effect of a given level of unemployment.

Both these mechanisms appear to be at work in the South African labour market, which is characterised by consistently high levels of unemployment, a severe lack of skills, large numbers of discouraged and unemployable workers and strong labour unions. Fortunately, the NAWRU seems to have peaked lately and has since trended lower, which could result in the future reversal of the observed unemployment rate. Such a reversal, however, will be short lived if not accompanied by adequate labour market reform.

4.3 Potential Output

Given the estimates for trend total factor productivity and potential employment, potential output can be calculated:

$$Y_t^* = A_t^* K_t^{\alpha} (L^*)_t^{1-\alpha}$$

with $\alpha = 0.301989$, A_t^* the HP-filtered series for total factor productivity and L_t^* potential employment.

Figure 3 shows the estimated output gap, calculated as the difference between actual and potential output, as a percentage of real GDP. Shaded areas represent official downswings in economic activity as recorded by the SARB.

Insert Figure 3 here

The estimated output gap matches the official turning points in all but two instances, with actual output increasing relative to potential output during upswings and vice versa during downswings.

Figure 4 compares the estimated output gap to two other measures: the output gap obtained by applying a simple HP-filter to actual output, and the output gap variable employed in the Bureau for Economic Research's annual forecasting model. All three output gap variables are expressed as a percentage of real GDP.

Insert Figure 4 here

The three different estimates of the output gap display broadly similar patterns across the sample, although the new estimate does imply a steeper decline in actual relative to potential output during the 1996 to 1999 downswing, while pointing towards an earlier recovery during the same period. An important improvement over the alternative estimates is the steeper growth in actual output relative to potential output during the 1999 to 2007 upswing phase. This was a period of unprecedented economic growth and it is highly plausible that the economy performed above potential for most of the period.

4.4 Can the South African economy grow at 7%?

The South African government has on numerous occasions indicated its intention to enact policies that would, if successful, stimulate economic growth (this includes the recently published New Growth Path) and has set a target of 7% for the economy as a whole. The question remains whether this target rate is reachable, not to mention sustainable, over the long term.

The results obtained for potential output based on the structural production function approach suggests that the potential sustainable growth rate (i.e. that level of output growth consistent with stable wage inflation) for South Africa is about 3.5%. This is substantially lower than the 7% target set by government.

Additionally, capacity utilisation appears to be fluctuating around 100%, which suggests that there is very little room for continued, sustainable expansion. The structural problems in the economy, especially in the labour market, contribute to the fact the South Africa’s potential to grow has been diminishing over time and that increases in output put pressure on prices and wages much faster than previously thought.

It appears as if the South African economy has hit capacity constraints, with potential growth far below official targets. Any policy aimed at increasing economic growth, while neglecting to address the various structural problems (specifically in the labour market) in the economy, will prove to be unsustainable. Labour market reforms and policies aimed at increasing the potential capacity of the South African economy are imperative in the search for sustainable, non-inflationary growth.

5. Conclusion

A Cobb-Douglas production function was used to obtain a new estimate of potential output for the South African economy. The underlying economic factors driving changes in potential output, namely growth in total factor productivity and potential employment, were discussed in turn.

Results indicate that the South African economy has a potential growth rate of around 3.5%, substantially lower than the 7% envisioned by government. It appears that the inflexible South African labour market, characterised by high unemployment, skills shortages and large numbers of discouraged workers, bears the brunt of the blame for the shortfall.

Government intervention, aggressive trade union behaviour, minimum wage requirements, excessive social security provision and inadequate labour skills all contribute to the inflexibility of the labour market. Broad reforms are required to ensure that the labour market regains its capacity to perform its allocative, informational and distributional functions in order for it to contribute in a meaningful way to economic growth and prosperity.

Recent discussions surrounding amendments to South African labour legislation do little to reassure that the South African government understands the structural impediments to higher growth emanating from the labour market, and might even be counterproductive in that inflexibility in the labour market could be reinforced. Labour market reforms and policies aimed at increasing the potential capacity of the South African economy are imperative in the search for sustainable, non-inflationary growth.

Appendix A: Data

Variable	Symbol	Source
Real GDP (constant 2005 prices)	Y	SARB
Actual Capital Stock	K	SARB
Total Employment	L	Quantec
Labour Force (economically active population)	LF	Quantec
Participation Rate	P	Quantec
Smoothed Labour Force	LFS = LF*(trend_P)	
Nominal Wage Bill	YWB	SARB
Consumption Deflator	PC	SARB
Real Wage Rate	W = (YWB/PC)/L	

Appendix B: The Kalman Filter[11]

The Kalman filter is a recursive procedure for computing the optimal estimate of the state (or unobserved) vector at time t, given the information set at time t, including information contained in the observed variables, and presents a linear estimation method for equations represented in state space.

Application of the Kalman filter requires the system to be written in state space form, with a **measurement equation**:

$$y_t = Z.X_t + R.D_t + e_t$$

where Z and R are vectors of parameters, X is a vector of unobserved variables (e.g. Total factor productivity in (1) in the text), and D is a vector of observed exogenous variables;

and a **transition equation**:[12]

$$X_t = T.X_{t-1} + \varepsilon_t$$

where e_t and ε_t are IID normally distributed with zero mean and variance $H_t = \sigma^2$ and $q_t = \sigma^2 Q$ respectively. The ratio $H_t/q_t = Q$ is called the signal-to-noise ratio[13]. T is a vector of parameters.

The Kalman filter is made up of two stages:

The **filtering procedure** builds up the estimate of the unobserved state as new information about the observed variable becomes available. If a_t is the optimal estimate of the state X_t , with P_t its variance/covariance matrix, then, given a_{t-1} and P_{t-1}, the Kalman filter may be written:[14]

[11] Standard references include: Cuthbertson, Hall and Taylor (1992), Harvey (1992) and Hamilton (1994)
[12] Various forms of the transition equation can be used. The most common is to use an AR or random walk specification, although other specifications are also used. The random walk specification is used for ease of exposition
[13] In applying the Kalman filter, initial values need to be specified for the variances of the error terms, and hence the signal-to-noise ratio.

$$a_{t+1|t} = (T - K_t Z)a_{t|t-1} + K_t(y_t - d_t)$$

and

$$P_{t+1|t} = T\left(P_{t|t-1} - P_{t|t-1}Z'F_t^{-1}ZP_{t|t-1}\right)T' + Q$$

with $K_t = TP_{t|t-1}Z'F_t^{-1}$ and $F_t = ZP_{t|t-1}Z + H_t$

These equations permit the calculation of the **prediction errors** for period t as:

$$v_t = y_t - Za_{t|t-1} - R.D_t$$

which go into the likelihood function:

$$l_t = -\frac{1}{2}\log 2\pi - \frac{1}{2}\log|F_t| - \frac{1}{2}v_t F_t^{-1} v_t$$

The series $\{a_t\}$ that maximises the likelihood function gives an optimal one sided estimate of the unobserved state.

The **smoothing procedure** uses information from the whole sample of observed variables. It is a backward recursion starting at time T and produces smoothed estimates in the order T,...,1 following the equations:

$$a_{t|T} = a_t + P_t^*\left(a_{t+1|T} - T_{t+1}a_t\right)$$

$$P_{t|T} = P_t + P_t^*(P_{t+1|T} - P_{t+1|t})P_t^{*\prime}$$

with $P_t^* = P_t T_{t+1}' P_{t+1|t}^{-1}$, $a_{T|T} = a_T$ and $P_{T|T} = P_T$

[14] The starting values a_0 and P_0 are important for the optimization process to converge and need to be specified. If the starting values are too far away from the true values, the system will not converge

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Table 1: Cobb-Douglas Production Function Estimation

Coefficients	**Estimates**
C(1)	0.301989*
C(2)	-31.95060
C(3)	-7.721510*
Final State	
SV1	3.250476*

Note: * indicates significance at the 5% level

Table 2: Contributions to Growth, 1971-2010

	1971-1993	1994-2010	1971-2010
Real GDP growth	2.01%	3.19%	2.51%
Contributions			
Labour	1.08%	0.78%	0.96%
Capital	1.03%	0.62%	0.86%
TFP	-0.11%	1.79%	0.70%

Source: Own calculations

Figure 1: Total factor productivity obtained from the Cobb-Douglas estimation

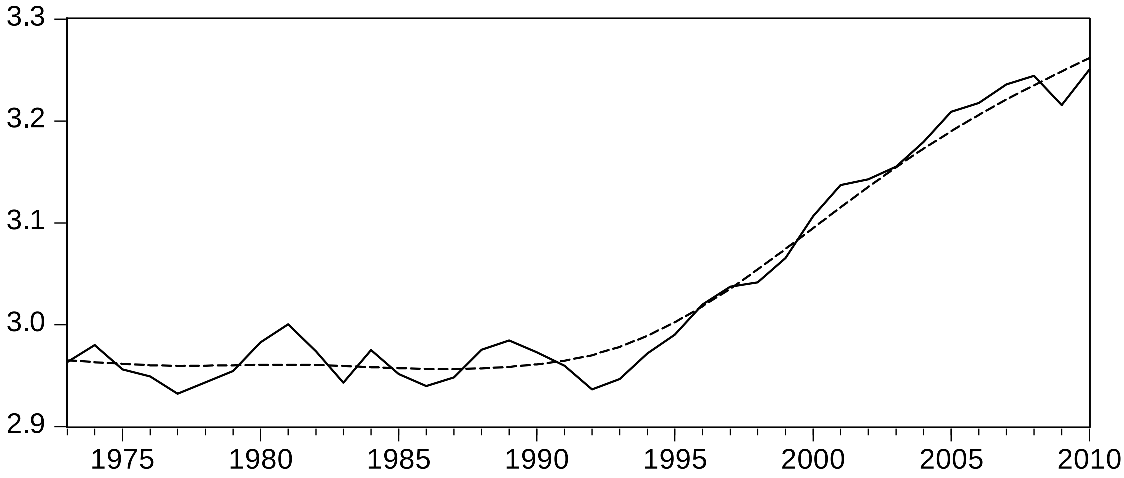

Figure 2: Non-accelerating Wage Rate of Unemployment (NAWRU)

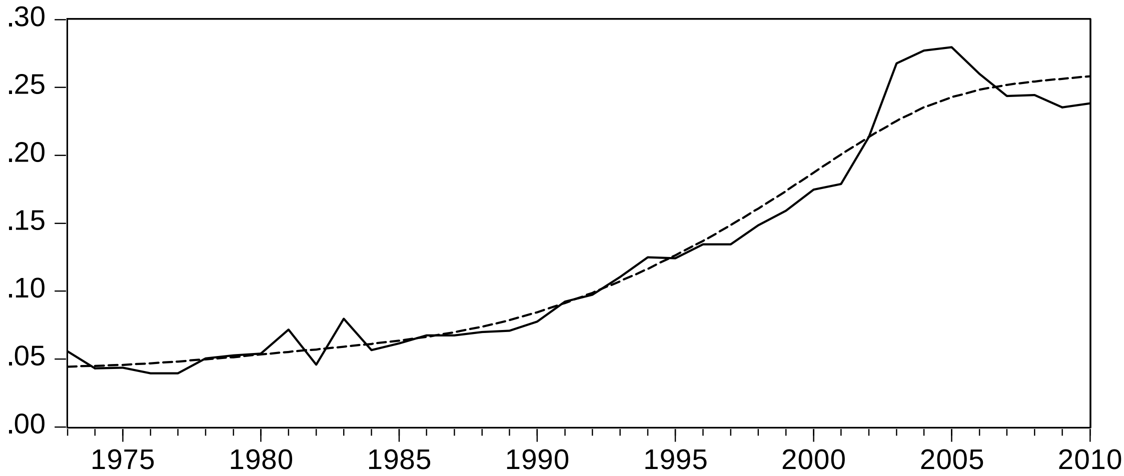

Figure 3: Estimated output gap

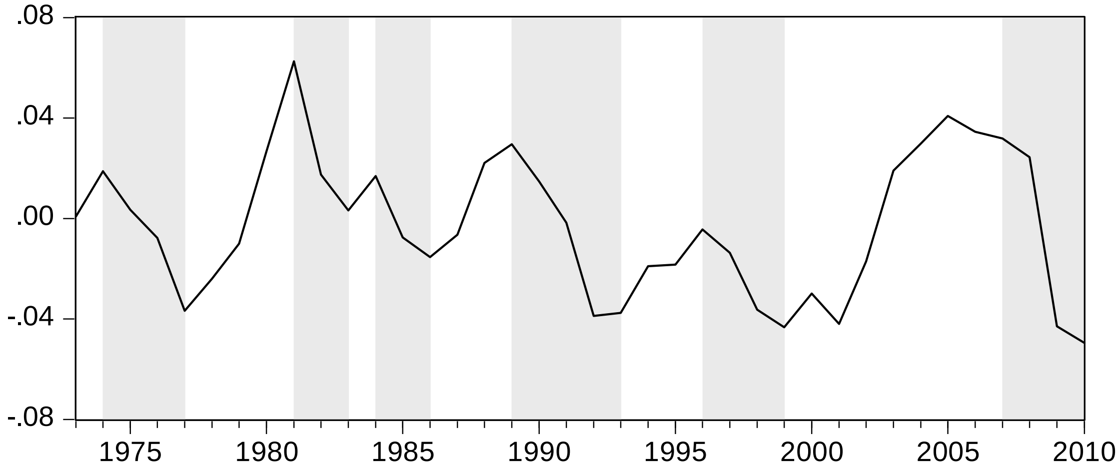

Figure 4: Estimated output gap: Comparison of different measures