THE SOUTH AFRICAN RISK-FREE RATE ANOMALY

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ABSTRACT

International tests of the suitability of the Capital Asset Pricing Model (CAPM) have found that the minimum return required by investors implied by the model exceeds the risk-free proxy rate. Similar tests in South Africa have found that the minimum required return is not significantly different from the risk-free proxy return. This study seeks to resolve this apparent anomaly by employing a direct and indirect approach to estimate the minimum return required by investors. It is found that, in keeping with international evidence, the minimum required rate of return exceeds that of the risk-free rate proxy; whilst the minimum-variance zero-beta portfolio return closely approximates the minimum required return. The implications of these findings for researchers and practitioners using the CAPM are discussed.

Keywords: Capital Asset Pricing Model, zero-beta portfolio, risk-free rate, required rate of return

JEL Codes: G11, G12, C12, C21

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1. INTRODUCTION

"The CAPM is a wonderful theory. It is also useless in a practical way" (Ross, 1993: 13). Since the development of the CAPM by Sharpe (1964) and Lintner (1965), it has been the subject of considerable debate and criticism, yet the model remains a fundamental tool in financial analysis in order to estimate the cost of equity for a firm (Graham and Harvey, 2001: 201; Correia and Cramer, 2008: 41). One of the difficulties in employing the CAPM in practice is the correct identification of the parameters of the model as they are all theoretical constructs that have to be estimated. When estimating the risk-free rate, short-term Treasury bills (T-Bills) and long-term Treasury bonds (T-Bonds) are generally employed as proxies for the risk-free rate.

Some of the earliest tests of the validity of the CAPM tested the Sharpe-Lintner hypothesis that the minimum required return implied by the observed Security Market Line (SML) should equal the risk-free rate proxy. Black *et al.* (1972), Fama and MacBeth (1973) and Stambaugh (1982) however, found that the minimum rate of return required by investors (observed by regressing actual returns against risk and then estimating the intercept of this regression) significantly exceeded the return of commonly employed risk-free rate proxies such as the United States (U.S.) one-month T-Bill rate. Subsequent tests both in the U.S. and internationally, including Faff (2001), Chou and Lin (2002) and Fama and French (2004) have continued to observe this relationship across both time and different markets.

Only two South African studies have attempted a similar analysis; Bradfield *et al*. (1988) found no significant difference between the minimum required return and the risk-free rate proxy, while van Rhijn (1994) found that the yield on the risk-free rate proxy *exceeded* the observed minimum required return. The existing empirical evidence thus seems to point to a fundamental difference in the risk-return relationship for the South African market. This paper seeks to address this apparent anomaly by employing both indirect and direct tests of the Sharpe-Lintner hypothesis. We find that, in keeping with international evidence, the minimum required return of investors exceeds the yields on the most common risk-free rate proxies employed in South Africa but is not significantly different from the minimum-variance zero-beta portfolio return. This result provides strong evidence in favour of employing Black's (1972) minimum-variance zero-beta portfolio approach in estimating the risk-free rate of return in applications of the CAPM and as a result our study has important implications for academics and practitioners using the CAPM.

The remainder of this paper is structured as follows: in Section 2 the international empirical tests of the Sharpe-Lintner hypothesis are discussed in conjunction with possible explanations for these findings, with a review of the South African studies provided in Section 3. In Section 4, the methodology followed in this study is detailed, with the results presented in Section 5. Conclusions and recommendations for further research are provided in Section 6.

2. INTERNATIONAL EVIDENCE: THE SHARPE-LINTNER HYPOTHESIS

While Sharpe's (1964) original CAPM assumed the existence of a risk-free asset, as early as 1972, Black demonstrated that this assumption was not necessary. In Black's version of the CAPM the return on a single risk-free security is replaced with the return on the minimum-variance zero-beta portfolio. Black (1972) argued that if a single risk-free asset does not exist, then the minimum-variance zero-beta portfolio is a more suitable measure of the risk-free rate. The validity of the use of the zero-beta portfolio as an alternative for the risk-free rate has been tested indirectly (a phrase first used by Morgan (1975: 363)) in the literature by estimating the implied minimum return required by investors, represented by the intercept of the SML, and comparing this estimate to the chosen risk-free rate proxy.

If the market risk premium is positive, the test of whether the minimum required return differs significantly from the risk-free rate proxy is a test of the suitability of the traditional CAPM (and hence is termed the Sharpe-Lintner hypothesis), whilst if the minimum return differs significantly from the risk-free rate proxy but not from the minimum variance zero-beta return, then it can be concluded that zero-beta approach is more suitable.

Black *et al.* (1972) and Fama and MacBeth (1973) were among the first to test the Sharpe-Lintner hypothesis as part of more comprehensive tests of the CAPM. In both studies, the null hypothesis that the estimated intercept was equal to the risk-free proxy (one-month T-Bills) was rejected as the intercept of the SML was found to exceed the risk-free proxy yields, as shown in Table 1. The conclusion of Black *et al.* (1972) and Fama and MacBeth (1973) therefore, was that the minimum-variance zero-beta portfolio provides a more accurate estimate of the minimum return required from investing, given that the other CAPM relationships held over the period examined.

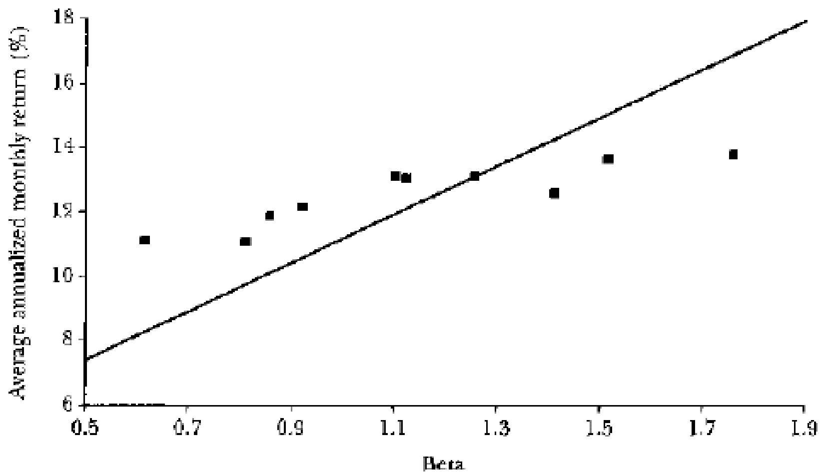
Table 1: Summary of International Studies

Study	Country	Period	Monthly Intercept Estimate (%)	T/Z Statistics ($\gamma_0 = R_f$)	Conclusion
Black *et al.* (1972)	U.S.	1931-1966	0.36	6.52	Significant ($\gamma_0 > R_f$)
Fama and MacBeth (1973)	U.S.	1935-June 1968	0.61	2.55	Significant ($\gamma_0 > R_f$)
Stambaugh (1982)	U.S.	February 1953-December 1976	0.12	4.82	Significant ($\gamma_0 > R_f$)
Fama and French (2004)	U.S.	1928-2003	0.77	Not given	Significant ($\gamma_0 > R_f$)
Fraser *et al.* (2004)	U.K.	1975-1996	0.86	0.43	Insignificant ($\gamma_0 > R_f$)
Faff (2001)	Australia	1974-1995	1.1	Not given	Significant ($\gamma_0 > R_f$)
Chou and Lin (2002)	International	1980-1997	1.63	Not given	Significant ($\gamma_0 > R_f$)

Stambaugh (1982) followed a similar approach to these studies, but employed Maximum Likelihood as the estimation technique rather than Ordinary Least Squares and broadened the sample to include preference shares and bonds. Despite this, Stambaugh (1982) reached the same conclusion regarding the relationship between the risk-free rate proxy and the intercept. More recently, Fama and French (2004) updated the analysis of Fama and MacBeth (1973) and confirmed that the intercept of the regression model exceeded the estimate of the risk-free proxy, as shown in Table 1. The results of the Fama and French (2004) study are also presented graphically in Figure 1, in conjunction with the theoretical SML based on the one-month T-Bill rate and the average market return. The actual CAPM relationship (shown by the points) was characterised by a higher intercept and a flatter slope than the theoretical relationship (the solid line). Thus, despite many textbooks advocating the use of the T-Bill as a suitable proxy for the risk-free rate, the empirical evidence clearly indicates that this rate understates the true minimum return that investors require from investing. In light of this evidence, some texts have advocated the use of a longer-term T-Bond for the risk-free proxy as the yield on longer-term instruments is likely to exceed that of short-term securities; but there is no evidence to support the claim that T-Bonds more closely approximate the minimum return.

Similar studies to these U.S. based analyses have been conducted in the U.K., Australia and in the international market. As can be seen from Table 1, the identical relationship to the U.S. was identified by Faff (2001) for Australia and Chou and Lin (2002) for the international market, with the minimum required returns exceeding the risk-free proxy yields. The one exception to these results arises in the study of Fraser *et al.* (2004) in the U.K., who found that although the minimum required return was greater than the one month T-Bill yield, the difference was not statistically significant. Therefore the conclusion reached was that in the U.K. government securities are considered suitable proxies to estimate the risk-free rate (Fraser *et al.*, 2004: 267-268).

Figure 1: Theoretical SML versus Actual SML 1928 – 2003



(Source: Fama and French, 2004: 33)

Morgan (1975) conducted a direct test of the validity of the zero-beta portfolio to estimate the risk-free rate in the U.S. Morgan (1975: 363-364) obtained estimates of the zero-beta portfolio returns by formulating an objective function for the variance of a portfolio subject to the constraints that the weights of the assets in the portfolio must sum to one and that the portfolio beta must equal zero. By minimising this objective function, Morgan (1975: 364) was able to solve for the matrix of weights of the individual assets in the minimum-variance zero-beta portfolio before then computing the portfolio returns. In addition to this, Morgan (1975: 362) estimated the implied minimum required return based on the methodology of Fama and MacBeth (1973). Rather than testing the statistical equality of the two estimates, Morgan (1975: 365) compared the predictive power of the CAPM using the minimum rate of return and the zero-beta portfolio return. His results showed that there was no statistical difference between the predictions using either the zero-beta portfolio return or the minimum required return and therefore concluded that the zero-beta portfolio approach is a suitable means to estimate the risk-free rate. However, Morgan (1975) did not consider the predictive power of the CAPM using a risk-free rate proxy to ascertain the robustness of his results.

The fact that short-term T-Bills have been found to understate the minimum required return does appear to be counterintuitive as it would be expected that if these proxies are not suitable they would overstate rather than understate the true minimum return. A number of possible explanations have been proposed to account for this finding, but the fact that this relationship is consistent across the majority of markets suggests that it is not a shortcoming associated with a particular market but rather a problem with the model or its application. For example, the historical-based tests of an expectational model may be incorrect, the form of the model tested may be inappropriate, the market-portfolio proxy may not be mean-variance efficient or the risk-free proxy may be unsuitable. However, a number of these explanations have been discarded on the basis of further studies, such as different forms of the model still signalling that the minimum return exceeds the T-Bill yield (Breeden *et al.*, 1989), or that the use of a mean-variance efficient proxy for the market portfolio still results in the same observation (Stambaugh, 1982).

One argument, however, which has not been empirically disproven, is that the risk-free proxy does not truly measure what was defined by Sharpe (1964) as the risk-free rate. T-Bills only represent a proxy for the riskless investment rate, but by definition, the riskless return should represent the rate at which investors can both borrow and lend. The rate at which investors can borrow exceeds this proxy rate and thus this may account for the disparity observed between the minimum return that investors require from investing and the proxies considered. It is precisely for this reason that Brennan (1971) advocated the use of the zero-beta portfolio rate as the measure of the risk-free rate given the difficulty in trying to determine the weighted average of the riskless borrowing and lending rates, as the CAPM calls for.

3. SOUTH AFRICAN EVIDENCE: THE SHARPE-LINTNER HYPOTHESIS

Bradfield *et al.* (1988) and van Rhijn (1994) have employed similar methodologies to Black *et al.* (1972) and Fama and MacBeth (1973) to test the validity of the CAPM, including the Sharpe-Lintner hypothesis, in South Africa. Bradfield *et al.* (1988: 14-15), using a sample of 100 shares, found that the fundamental relationship between risk and return postulated by the CAPM held for shares on the Johannesburg Securities Exchange (JSE). However, they identified no statistically significant difference between the minimum required return and the twelve month fixed deposit rate, used as a proxy for the risk-free rate; although in four of the five periods the intercept was greater than the proxy. Accordingly, Bradfield *et al.* (1988: 15) concluded that a proxy is more suitable than the zero-beta portfolio as the risk-free rate in the CAPM in South Africa. This result is similar to that of Fraser *et al.* (2004) for the U.K.

In interpreting the findings of Bradfield *et al.* (1988) however, it must be noted that the choice of proxy is unusual. Bradfield *et al.* (1988) do not provide any justification for their choice but it may be related to the deficiencies associated with the South African proxies. The fixed deposit rate, determined by a commercial bank, is likely to exceed the rate on more traditional government proxies and thus may have contributed to the differences observed between the South African and U.S. studies.

Similarly to Bradfield *et al.* (1988), the results of the test of the Sharpe-Lintner hypothesis of van Rhijn (1994) do not conform to those referred to in the preceding section. The average monthly estimate of the three-year government bond used as a proxy for the risk-free rate was 1.08%, whereas the estimated intercept was only 0.92% per month (van Rhijn, 1994: 322). Due to insufficient data (only industrial shares on the JSE were used in the study), van Rhijn (1994: 322) was unable to test whether the difference was statistically significant; concluding simply that the difference was small. This conclusion would appear to support the finding of Bradfield *et al* (1988); noticeably however, in contrast to all other evidence he finds that the intercept is lower than the risk-free rate proxy.

The two South African tests of the Sharpe-Lintner hypothesis thus do not conform to the international evidence regarding the relationship between the minimum required return and the risk-free rate proxy. Given that the reintegration of South Africa into the global economy post 1994 has resulted in considerable development of the market and that it is now possible to perform a more sophisticated direct test of the Sharpe-Lintner hypothesis, it is of value to revisit this apparent anomaly.

4. METHODOLOGY

4.1 Hypothesis Tests

In order to comprehensively test the Sharpe-Lintner hypothesis, two approaches were used. The indirect method, which has predominantly been followed in the international studies and in the South African research, entails determining whether the risk-free rate proxy yield was equal to the historical minimum required return. The hypothesis that is tested for this purpose is shown in equation 1. If the null hypothesis is rejected, it can be concluded that the use of a proxy for the risk-free rate is not appropriate; but it is only possible to conclude that the minimum-variance zero-beta portfolio is preferable if the direct test of the Sharpe-Lintner hypothesis is also conducted. This entails comparing the minimum-variance zero-beta portfolio returns to the minimum required return, as shown in equation 2. If the null hypothesis is rejected in test 1 but is not rejected in test 2, this provides strong evidence against the use of a proxy for the risk-free rate and support for Black's (1972) minimum-variance zero-beta portfolio returns.

$$H_0: \gamma_0 = \bar{R}_f \quad \text{and} \quad H_1: \gamma_0 \neq \bar{R}_f \tag{1}$$

$$H_0: \gamma_0 = \bar{R}_z \quad \text{and} \quad H_1: \gamma_0 \neq \bar{R}_z \tag{2}$$

Where: γ_0 is the implied minimum required rate of return, $\bar{R}_f$ is the return on the risk-free rate proxy and $\bar{R}_z$ is the return on the minimum-variance zero-beta portfolio.

A t-test of the equality of means was used to examine both of these hypotheses as this formula allows for variation in both the estimated coefficient and the hypothesised value. The method followed to estimate the minimum required return and the zero-beta portfolios returns, and the proxies used for the risk-free rate are discussed in the following sections.

4.2 Estimation of the Minimum Required Return

Given an estimate of the risk-free rate and the market return the SML can be derived, which then indicates the return that a share should earn given its beta. The Black *et al.* (1972) and Fama and MacBeth (1973) methods of testing the CAPM essentially work backwards from this approach, by obtaining betas and returns for shares and then computing the SML implied by these securities. The equation commonly used for this purpose is depicted in equation 3. The slope of the SML provides an estimate of the actual market risk premium, with the intercept representing the minimum return required by investors (Cochrane, 2001: 236).

$$R_i = \gamma_0 + \gamma_1 \hat{\beta}_i + \varepsilon_i \quad (3)$$

Where: R_i are the returns on shares i = 1, 2 … 3, $\hat{\beta}_i$ are the betas of the shares, and ε_i are the residuals of the regression.

In order to estimate the SML, risk and return values are required for all assets in the market. Monthly prices and dividend yields were obtained from the JSE for all shares over the period 1993-2008. The number of shares was adjusted each year for any new listings or de-listings. The share returns were calculated as the sum of the capital gain and dividend yields.

In equation 3, share betas are used as the explanatory variable; however, betas of individual shares tend to be volatile (Cuthbertson and Nitzsche, 2005: 191). Accordingly, tests of the CAPM have combined shares into portfolios and either estimated equation 1 using only the portfolios (as per Black *et al.*, 1972) or using the individual shares but allocating the appropriate portfolio beta to each share (as per Fama and MacBeth, 1973). Traditionally shares have been allocated to portfolios based on historical beta estimates in order to achieve a wide dispersion in beta values (Fama and French, 2004: 36); but this necessitates assuming that historical estimates of beta are good predictors of future values. Sorting the shares based on industry classifications has been proposed as an alternative (Lewellen *et al.*, 2010: 186), as it achieves a similar distribution of betas but without relying on the inference that historical betas are good predictors of future values. Both of the South African studies have relied on the beta-sorting procedure and thus, it is possible that this method of sorting may have biased the results if the underlying assumption is inappropriate. Consequently, industry-sorting was used based on the industry classifications shown in Table 2.

Table 2: Industry Classifications

Building and Construction	Basic Resources	Media	Food and Beverage
Technology and Electronics	Insurance	Gold Mining	Retail
Personal and Household Goods	Industrial Engineering	General Mining	Travel and Leisure
Chemicals, Oil and Gas	Industrial Transportation	Other Mining	Automobiles and Parts
Banks and Financial Services	Other Industrial	Property	Health Care

The shares were equally-weighted within each portfolio to obtain the average monthly portfolio returns. Thereafter, the beta of each portfolio was determined by regressing the portfolio returns against the market portfolio returns over the period 1993-2008 (and across sub-periods 1993-2000, 2001-2008, 1993-1997, 1998-2002 and 2003-2007), as per equation 4.

$$R_{pt} = \alpha_0 + \beta_p R_{mt} + \eta_t \quad (4)$$

Where: R_{pt} are the returns on portfolio p over time t, α_0 represents the excess returns earned on the portfolio, β_p represents the beta of each portfolio and R_{mt} are the returns on the market portfolio proxy over time t. The All Share Index (ALSI) was used as the proxy for the market portfolio in this regression. Although the use of the ALSI for this purpose has been criticised (see for example van Rensburg, 2002 and Correia and Uliana, 2004), it remains the most commonly used proxy in estimating betas (PWC, 2010: 31). The betas estimated for each portfolio from this regression were then used as the cross-sectional explanatory variable in equation 3 for each time period, as per Black *et al.* (1972).

As mentioned in Section 2, Stambaugh (1982) included preference shares and bonds in his study to ascertain whether the inferences of the CAPM were affected by the inclusion of other assets. In order to ensure the robustness of the results of this study, the analysis outlined above was repeated but including two preference share and three bond portfolios. The preference shares were allocated to the portfolios based on historical betas as the industry affiliations of the securities were too broad to split the shares into two. The beta for each share was obtained for each year as the slope coefficient from a time-series regression of the returns of the share on the ALSI using the immediately preceding 36 months of data (at least 24 months), and the composition of the portfolios was adjusted each year to allow for changes in betas and any newly listed or delisted shares. The Bond Exchange of South Africa indices for short-term (one- to three-years), medium-term (seven- to twelve-years), and long-term (twelve- to 30-years) bonds were used to represent the bond portfolios. As these bond indices were only formed in 2001, the estimates of the minimum required return including the additional asset classes were only obtained for the periods 2001-2008 and 2003-2007.

4.3 Estimation of the Zero-Beta Portfolio Returns

Using the same portfolios, the minimum-variance zero-beta portfolio returns were computed over identical time horizons as the minimum required return, following the approach of Morgan (1975). The weightings of the various asset portfolios in the zero-beta portfolio were determined by minimising the variance of the zero-beta portfolio subject to the constraints that the sum of the weightings of the individual portfolios must equal one and the weighted average of the portfolio betas must equal zero, as shown in equations 5-7. Microsoft Excel's Solver function was then used to solve this set of equations.

$$\sigma^2_z = \sum_{q=1}^{N} \sum_{p=1}^{N} x_{zq}\, x_{zp}\, \sigma_{qp} \tag{5}$$

$$\sum_{p=1}^{N} x_{zp} = 1 \tag{6}$$

$$\sum_{p=1}^{N} x_{zp}\, \hat{\beta}_p = 0 \tag{7}$$

Where: σ^2_z is the variance of the zero-beta portfolio, x_{zq} and x_{zp} are the weightings of portfolios p and q in the zero-beta portfolio, σ_{qp} is the covariance in the returns of portfolios p and q, and p and q represent the individual portfolios. Once the portfolio weights were obtained, the minimum-variance zero-beta portfolio returns were computed as follows:

$$R_z = \sum_{p=1}^{N} x_{zp}\, R_p \tag{8}$$

4.4 The Risk-Free Rate Proxies

Both T-Bonds and T-Bills were selected as proxies for the risk-free rate to ensure that the results of these tests were robust to the choice of proxy. Thus this study extends the international literature by ascertaining whether a longer-term T-Bond more closely approximates the minimum required return as suggested by several texts (Copeland *et al.*, 2000; Brigham and Ehrhardt, 2005). The data on the three-month T-Bill was obtained from the South African Reserve Bank and the R157 yield, used to represent the T-Bond, was obtained from McGregor's BFA. The average monthly yield on the three-month T-Bill was computed by dividing the equivalent annual rates by twelve; whilst for the R157 government bond, the annual percentage rate was first determined (assuming monthly compounding) before dividing this value by twelve to obtain the monthly average.

5. RESULTS AND ANALYSIS

The values of the intercept and slope coefficients from equation 3 (using only ordinary shares) and the standard errors thereof are shown in panel A of Table 3. The direct estimates of the minimum-variance zero-beta portfolio returns and the standard deviation associated with these returns are also presented. The estimates of γ_1 are positive and statistically significant for the entire period and for four of the five sub-periods and thus provide support in favour of the assumption that the CAPM is a suitable tool to employ in analysing South African shares. The negative slope coefficient for 1998-2002 corresponds to a period in which three distinct contractions in the market (1998, 2000 and 2002) occurred, and thus this finding is not in complete disparity with theory. In panel B of Table 3 the regression coefficients and the zero-beta portfolio returns are shown for the periods where the bond and preference share portfolios were also included in the sample. The inclusion of these asset classes results in a larger and more significant estimate of the market risk premium.

Table 3: Coefficient Estimates and Zero-Beta Portfolio Returns

Period	γ_0	**Standard Error**	γ_1	**Standard Error**	R_z	**Standard Deviation**
	A: Estimates using ordinary shares only					
1993-2008	1.7019***	0.6924	0.825*	0.4325	1.6133*	0.7544
1993-1997	1.6583***	0.4734	1.1619*	0.6069	1.6570***	0.5900
1998-2002	4.0301***	1.1789	-3.8047*	1.9367	5.3657***	0.8687
2003-2007	2.7085***	0.522	0.3673**	0.1524	2.7082***	0.6432
1993-2000	1.8587***	0.5166	1.3391**	0.6719	1.6366***	0.6594
2001-2008	1.6196***	0.2585	0.217*	0.1159	1.5615***	0.4116
	B: Estimates using ordinary shares, preference shares and bonds					
2003-2007	1.6125***	0.3436	2.0870***	0.6563	1.6130***	0.3928
2001-2008	1.3550***	0.2166	1.3105***	0.3789	1.2499***	0.3704

***, **, * Statistically significant at the 1%, 5% and 10% level respectively

All of the intercept estimates are statistically significant; with the inclusion of preference shares and bonds having no visible impact on the intercepts except to reduce the estimates of the standard errors. This finding is in keeping with the conclusions of Stambaugh (1982) that the inclusion of preference shares and bonds increases the efficiency of the coefficient estimates because these two asset classes tend to exhibit lower risk and therefore have lower betas than ordinary shares. Consequently, these observations lie closer to the vertical axis (i.e. smaller beta values), and thus the estimates of the intercept should be more accurate.

5.1 Hypothesis One

The test statistics, shown in Table 4, for the test of hypothesis one were significant at the one percent level for all periods using both the T-Bill and T-Bond; thereby indicating that the risk-free proxy yields differed significantly from the minimum return required by investors. All of the statistics were positive demonstrating that the risk-free rate proxy returns understated the minimum required return.

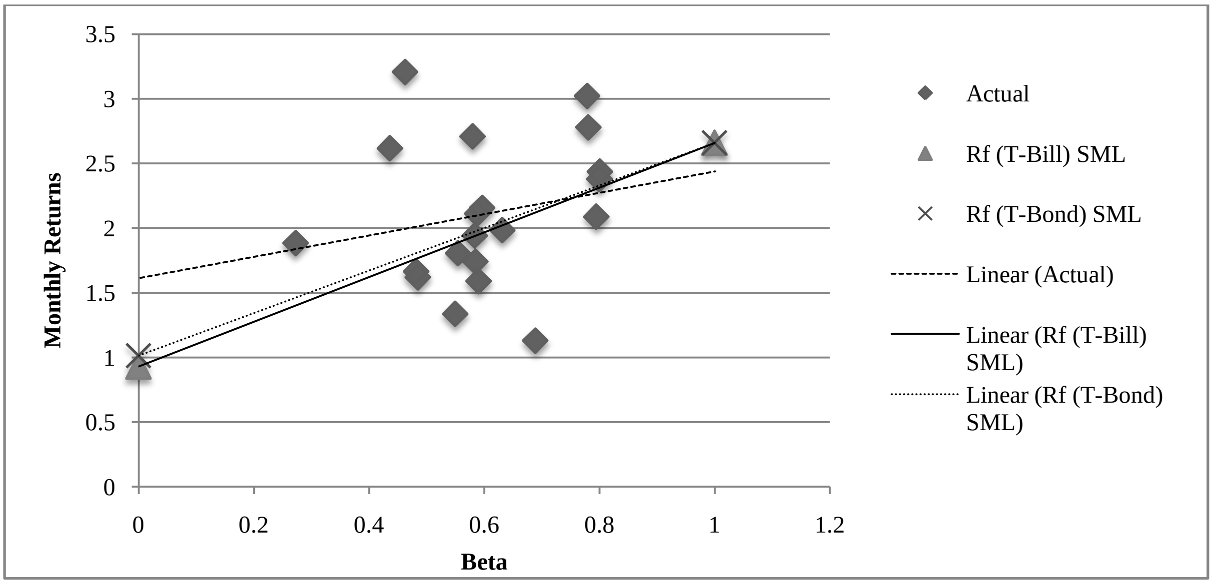
Table 4: Hypothesis Test Results

Period	**Hypothesis Test One**		**Hypothesis Test Two**
	T-Bills	**T-Bonds**	
	Estimates with ordinary shares only		
1993-2008	4.9511***	4.4104***	0.3870
1993-1997	5.1670***	3.7231***	0.0077
1998-2002	11.3870***	11.0405***	-4.0788***
2003-2007	17.0929***	16.9735***	0.0016
1993-2000	6.4791***	5.2963***	1.1857
2001-2008	14.3150***	14.1352***	0.5346
	Estimates with ordinary shares, preference shares and bonds		
2003-2007	12.9873***	12.8987***	-0.0043
2001-2008	12.8651***	12.6750***	1.0954

***, **, * Statistically significant at the 1%, 5% and 10% level respectively

Figure 2 depicts the actual relationship between risk and return based on the ordinary share portfolios and the implied SML, as well as the theoretical SML using both the T-Bill and T-Bond returns for the entire period 1993-2008. The difference between the values of the T-Bill and T-Bond returns are largely inconsequential, with the SML based on the T-Bond lying slightly above that of the SML with the T-Bill. The graph clearly indicates that the intercept implied by the risk-return relationships of the portfolios is significantly higher than either the T-Bill or T-Bond yields and consequently, the slope of the SML is much flatter than predicted by theory. This figure closely resembles that shown in Figure 1 from the Fama and French (2004) study. Thus, the results from this analysis conform to international studies that the minimum required return exceeds the risk-free proxy yields. The relationship between the proxy yields and the intercept of the SML is consistent with the relationship identified by Bradfield *et al.* (1988); but the difference is identified to be statistically significant in this study, which was not the case for Bradfield *et al.* (1988). As mentioned in Section 3, this difference could possibly be attributed to both the limited sample employed by Bradfield *et* *al.* (1988), the time period over which the study was conducted, the method[1] or the choice of risk-free rate proxy. The results however, contradict the findings of van Rhijn (1994) who identified the opposite relationship between the minimum required return and the risk-free proxy yield.

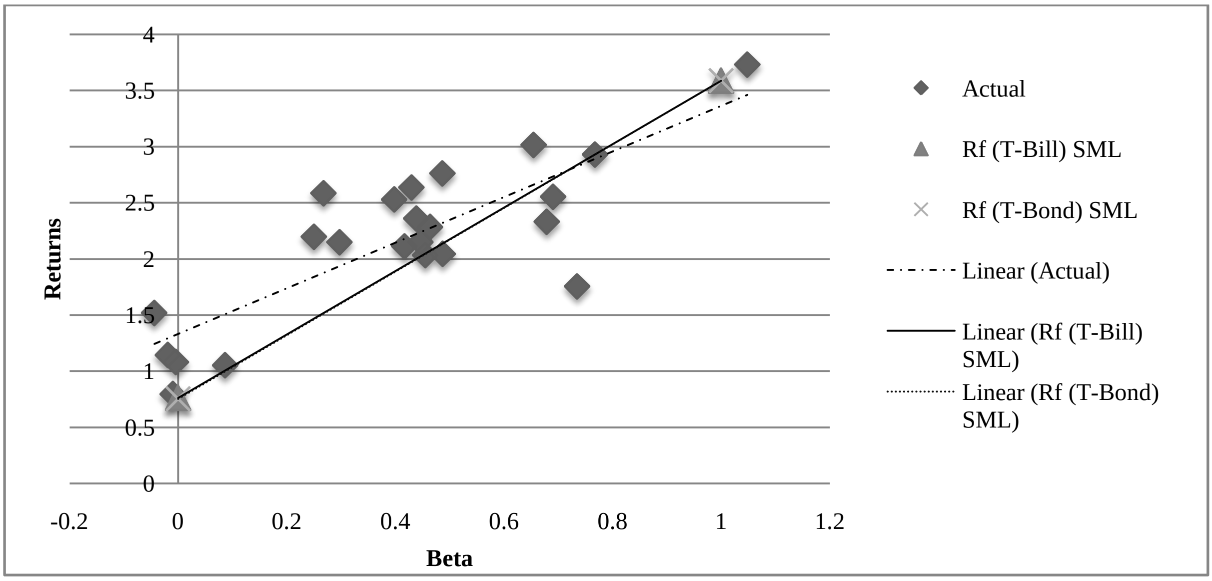
Figure 2: Actual Versus Theoretical SML (based on risk-free rate proxies) using Ordinary Shares, 1993-2008



In Figure 3, the actual and theoretical SMLs are plotted for the period 2001-2008 including the preference share and bond portfolios as well as the ordinary share portfolios. As is clearly evident, the inclusion of these two additional asset classes has no impact on the relationship between the risk-free rate proxy yields and the minimum required return. Thus, the expansion of the analysis to include preference shares and bonds reveals that the results of this study are robust to the choice of asset class and sample. As mentioned previously, it is apparent from Figure 3 that the preference share and bond portfolios are clustered around the zero-beta mark on the horizontal axis; thereby contributing to the more efficient intercept estimates obtained through the inclusion of these asset classes.

[1] The analysis in this study was repeated using ordinary share portfolios formed based on the beta-sorting procedure using the immediately preceding 36 months (at least 24 months) to estimate historical betas for each share, which were then used to rank the shares and allocate to portfolios. The results obtained do not differ from those reported here and thus it can be concluded that the different results obtained in this study compared to Bradfield *et a.l* (1988) cannot be attributed to the differences in asset allocation procedures used.

Figure 3: **Actual Versus Theoretical SML (based on risk-free rate proxies) using All Asset Classes, 2001-2008**



In Table 1 (page 4) the estimated values for the intercept of the SML obtained in several international studies were shown and although none of these studies examine the same period as was considered in this research, comparing the results enables some general conclusions to be drawn. The estimate of the monthly average intercept obtained for the period 1993-2008 was 1.7019, as shown in Table 3. This value more closely approximates those estimated by Faff (2001) for Australia, and Chou and Lin (2002) for the international market than the study of the U.S. by Fama and French (2004). This is not entirely unexpected given the developing status of the South African economy compared to that of the U.S. These results therefore appear to suggest that in countries which may be perceived to be of greater risk, investors require a larger base return from investing; in light of the discussions presented by Grandes *et al.* (2003: 8) and Hearn and Piesse (2009: 46) this conclusion conforms to expectations.

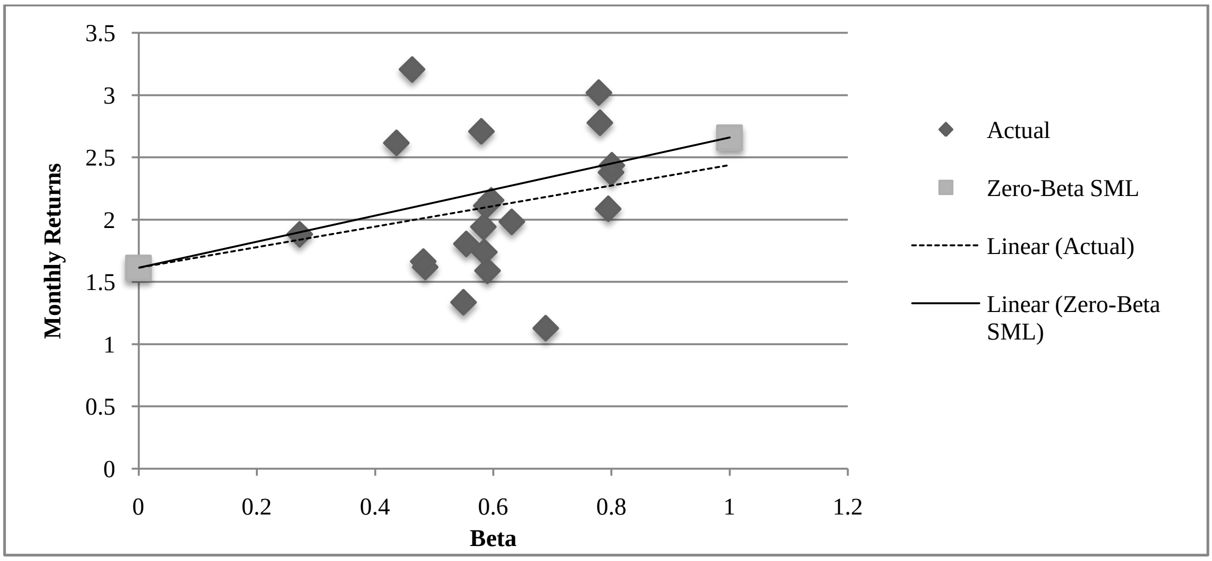
The surprising fact therefore is that despite the high interest rates observed in South Africa, the yields on government securities still understate the intercept of the CAPM, and in many instances the differences are more substantial than in the U.S., Australia and the international market as reflected by the higher t-statistics. The other similarity in the results obtained in this study to international research (Black *et al.*, 1972; Fama and MacBeth, 1973; and Stambaugh, 1982) is that the minimum required return fluctuates over time.

5.2 Hypothesis Two

The statistics computed for the second hypothesis, also shown in Table 4, are statistically insignificant for all periods, except 1998 to 2002[2]; thereby revealing that the minimum-variance zero-beta portfolio returns, in general, were not statistically significantly different from the minimum required return. This result mirrors the findings of Morgan (1975), discussed in Section 2.

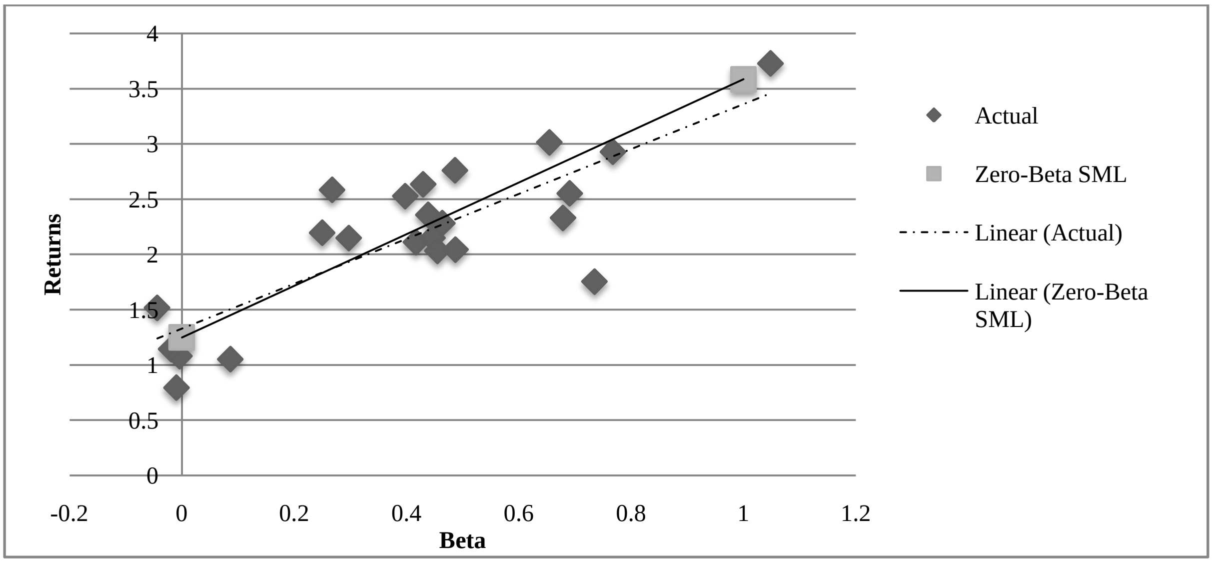
Figure 4 shows the same data points and implied SML as Figure 2 for the period 1993-2008; however, the theoretical SML is based on the minimum-variance zero-beta portfolio return rather than the risk-free proxy. This graph confirms that it is not possible to distinguish between the estimated intercept and the zero-beta portfolio return; that is, the zero-beta portfolio return provides a good estimate of the minimum required return. It is also evident that even with the use of the zero-beta portfolio return, the market risk premia are not identical (the slopes of the SMLs differ), but these differences are not statistically significant. In addition to this, as can be seen from panel B of Table 4 and Figure 5, the results of hypothesis two are also robust to the inclusion of preference shares and bonds in the sample.

Figure 4: Actual Versus Theoretical SML (based on the zero-beta portfolio) using Ordinary Shares, 1993-2008



[2] This discrepancy may be a result of the disruption to the risk-return relationship due to the Asian crisis.

Figure 5: Actual Versus Theoretical SML (based on the zero-beta portfolio) using All Asset Classes, 2001-2008



The results of hypothesis one and two thus infer that the use of a proxy for the risk-free rate is not appropriate in South Africa and that the minimum-variance zero-beta portfolio returns should be considered as preferable measure of the risk-free rate.

6. CONCLUSIONS AND IMPLICATIONS

In the majority of international studies, it has been documented that the short-term T-Bill understates the minimum required return. However, in two previous South Africa studies by Bradfield *et al.* (1988) and van Rhijn (1994) contrasting results were obtained, with the proxy not significantly understating the minimum return in the first and with the proxy actually overstating the minimum return in the second. This paper thus sought to resolve this South African risk-free rate anomaly by conducting an updated test of the relationship between the implied minimum return and the most commonly used proxies, as well as to provide a more direct test of the suitability of the minimum-variance zero-beta portfolio returns.

A more comprehensive sample and a longer time horizon were considered compared to the previous South African analyses and a different approach was also employed to allocate shares to portfolio. The results revealed that the risk-free proxy yields, similarly to international studies, understate the true minimum return required by investors whilst the minimum-variance zero-beta portfolio returns closely approximated the intercept of the SML over the period 1993-2008.

These findings thus reveal that the relationship between the risk-free proxies and the minimum required return does not differ in South Africa, contradicting previous South Africa research. The difference in findings may be as a result of limitations in the sample sizes used in previous studies, limitations in the methodology employed or simply the fact that the South African market has changed structurally since 1994. The fact that the minimum return exceeds the yields on T-Bills and the R157 government bond suggests that these instruments are not the best proxies for the risk-free rate. Further research, however, is required to quantify the impact of the incorrect specification of the risk-free rate in the CAPM on the reliability of the cost of equity estimates calculated. The finding that the minimum-variance zero-beta portfolio provides a better estimate of investors' actual minimum required rate of return has important implications for all practitioners and researchers who use the CAPM and indicates that Black's zero-beta approach may be more appropriate than the current practise of relying on government securities as proxies for the risk-free rate.

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