Rationalizable information equilibria*

Alexander Zimper†

August 9, 2011

Abstract

According to Radner's (1979) definition of a *rational expectations equilibrium* (=REE) every agent maximizes his expected utility with respect to his private information augmented with the information revealed through a common forecast function that becomes the equilibrium price function. This paper introduces the alternative general equilibrium concept of a *rationalizable information equilibrium* (=RIE) according to which every agent infers information from the values of market variables under the assumption that the agents' respective expected utility maximization problems are common knowledge between them. We prove that the RIE concept is a strict refinement of the REE concept that may exclude implausible REE.

Keywords: Rational Expectations, Walrasian Equilibrium, Asymmetric Information, Rationalizability

JEL Classification Numbers: D50; D83

*I am grateful to Roger Guesnerie, Hans Haller, Alex Ludwig, Herve Moulin, Klaus Ritzberger, and Nicholas Yannelis for valuable comments and suggestions.

†School of Economic and Business Sciences, University of the Witwatersrand, Johannesburg, South Africa. E-mail: alexander.zimper@wits.ac.za

1 Introduction

The general equilibrium concept of a *rational expectations equilibrium* (=REE) (Radner 1979) is motivated through the assumption of highly sophisticated agents who infer as much information as possible from observed values of market-variables such as prices or demand-supply decisions. In the words of Milgrom and Stokey (1982):

> "In rational expectations models it is assumed that each agent infers whatever information he can from the market variables he observes, as well as from the non-market signals to which he has access. Furthermore, in these models each agent believes—and is justified in believing—that all other agents also make full use of the information to them." (Milgrom and Stokey 1982, p. 19)

Formally, Radner (1979) defines a REE in terms of a so-called *forecast function* which can be interpreted as the agents' common price prediction for future states of a given economy. More precisely, a REE is defined through the consistency condition that some forecast function coincides with an equilibrium price function that clears markets in every state of the world such that each agent bases his demand-supply decision on his private information augmented with the information revealed through this forecast function.

At a superficial glance, the formal definition of a REE seems to capture the notion of highly sophisticated agents rather well: Every agent bases his decision on some ex-ante understanding of the economy's state-price generating process which turns out to be correct ex-post in the sense that predicted and actually implemented state-prices coincide. At closer inspection, however, the formal definition of a REE may give rise to implausible equilibria which are at odds with the notion of highly sophisticated agents who infer information from the observations at hand by means of an active reasoning process.

> **Example.** Consider the situation of an "insider" who offers to sell an asset to a potential investor at a high price. Further assume that this high price is only fair in the "good" state but too high in the "bad" state of the world whereby the actual state of the world is known to the insider but not to the investor. One REE has the insider selling the asset at fair prices, i.e., at a high price in the good and at a low price in the bad state, because the investor's self-fulfilling forecast function ensures him that he is actually in the good (bad) state whenever he observes the high (low) price.

By construction, the self-fulfilling forecast function of a REE excludes in the above example the possibility of a disappointed investor who learns ex-post that he had been tricked into a bad deal by the insider who sold him the asset at the high price in the bad rather than in the good state. While a REE thus rules out the possibility of ex-post disappointment by mere definition, there is no market institution in this set-up that actually protects the potential investor from possible disappointment. In a way, the self-fulfilling forecast function of a fully revealing REE amounts to the institution of a state-knowing Walrasian auctioneer who truthfully announces states of the world along with market-clearing prices for each state. Since every market participant can then rely on the truthfulness of these announcements, it becomes possible that the investor buys in the above example the asset at a high price from the insider. Arguably, such an equilibrium is implausible since it is at odds with the assumption of a highly sophisticated investor who anticipates the possibility that the insider might trick him when offering the asset at the high price.

In order to get rid of such implausible REE, the present paper introduces the general equilibrium concept of a *rationalizable information equilibrium* (=RIE) which explicitly formalizes the reasoning process of highly sophisticated agents in a general equilibrium framework under asymmetric information. In analogy to the pioneering game-theoretic work by Bernheim (1984), Moulin (1984)[1], and Pearce (1984), *rationalizability* refers here to a situation in which it is common knowledge between agents that they maximize their utility with respect to the beliefs they hold. More precisely, for any given observation of market prices and demand-supply decisions, I define an agent's *rationalizable information* as the information he infers under the assumptions that (i) every agent interprets observed demand-supply decisions at given prices as utility maximizing behavior, and (ii) every agent knows that the other agents interpret his demand-supply decision at given prices as utility maximizing behavior, and so forth. In a next step, I define a RIE through a market clearing price function such that each agent bases his demand-supply decision on his private information augmented with this rationalizable information. As this paper's main formal result I prove that every fully revealing RIE can be equivalently described as a fully revealing REE whereas the converse statement is not true. To see the intuition behind this refinement result consider again the above example.

> **The example reconsidered.** If the insider offers to sell the asset at a high price, the sophisticated investor of our model knows that such values of the market variables 'supply' and 'price' are consistent with the insider's util-

[1]While Moulin (1984) does not use the term *rationalizability*, his seminal analysis of best reply dynamics, on the one hand, and the iterated elimination of strictly dominated strategies, on the other hand, establishes—although restricted to the class of so-called *nice games*—many important technical properties of rationalizability- and adaptive learning concepts.

ity maximizing behavior in both states of the world. That is, the investor's rationalizable information—roughly speaking, the information he learnt from combining his knowledge about the insider's utility maximizing behavior with the observed values of market variables—does not allow him to distinguish between the good and the bad state. Consequently, the REE, in which the insider sells the asset at fair prices in any state of the world, is not a RIE whenever the investor attaches a strictly positive probability to the "bad" state, (for a detailed formal argument see Section 5).

Formally, the RIE concept may exclude a REE whenever the rationalizable information at REE values does not allow to pin down the true state of the world. Since the agents' rationalizable information at REE values gives rise to (weakly) coarser information partitions than a fully revealing REE price function, agents may have to solve different utility maximization problems under the RIE than under the REE concept whenever some agent's information partition is strictly coarser. According to the RIE concept such situations may only arise if some agent has the same optimal demand-supply decision—at fixed prices—in different states of the world so that other agents cannot learn from his demand-supply decision the true state of the world. Exactly this reasoning drives the above example: At a high price the insider would like to sell the asset in the "good" as well as in the "bad" state to the effect that the potential investor cannot rationalize from the insider's supply-decision whether he is in the "good" or in the "bad" state.

The remainder of this analysis is structured as follows. Section 2 formally defines the exchange economies we are concerned with. Section 3 recalls Radner's (1979) definition of a REE and Section 4 introduces the new RIE concept. Section 5 analyses in detail the above example, i.e., "The insider problem", whereas Section 6 discusses another example, i.e., "The lemon market problem", introduced by Mas-Colell, Whinston and Green (1995). The relationship of our approach to the existing rational expectations literature is discussed in Section 7. Finally, Section 8 concludes.

2 Exchange economies with asymmetric information

Consider a finite state space Ω with generic element ω that completely describes all aspects of uncertainty relevant to the economy's agents $i \in \{1, ..., n\}$.[2] Denote by Π_i an

[2] At this level of generality it does not matter whether Ω is interpreted as an "exogenous" or an "endogenous" or a hybrid state space (cf. Section 6).

arbitrary information-partition of agent i with *information cell* I_i as generic element. We write $I_i(\omega)$ for the unique element $I_i \in \Pi_i$ such that $\omega \in I_i$. Denote by $\Sigma(\Pi_i)$ the σ-algebra generated by Π_i and define the probability space $(\pi_i, \Omega, \Sigma(\Pi_i))$ such that the additive probability measure π_i stands in for agent i' subjective beliefs whereby we assume that $\pi_i(\omega) > 0$ for all $\omega \in \Pi$. Finally, we write $\langle \Pi, \pi \rangle$ where $\Pi = (\Pi_1, ..., \Pi_n)$ and $\pi = (\pi_1, ..., \pi_n)$.

We assume that all agents are expected utility maximizers such that $u_i : \Gamma_i \times \Omega \to \mathbb{R}$ stands for the utility that agent i obtains in state $\omega \in \Omega$ from the net-trade $\theta_i \in \Gamma_i \subseteq \mathbb{R}^l$ on some l-dimensional commodity space. That is, every agent i's plan of optimal supply-demand decisions at every information cell $I_i \in \Pi_i$ is an $\Sigma(\Pi_i)$-measurable mapping $\Theta_i : \Omega \to \Gamma_i$ that solves the maximization problem

$$\begin{aligned} & \max_{\Theta_i:\Omega\to\Gamma_i} E\left[u_i\left(\Theta_i(\omega), \omega\right), \pi_i(\omega)\right] \\ = & \sum_{I_i\in\Pi_i} \pi_i(I_i) \cdot \max_{\theta_i\in\Gamma_i} E\left[u_i(\theta_i, \omega), \pi_i(\omega \mid I_i)\right] \end{aligned} \tag{1}$$

subject to the budget constraint $P(\omega)\Theta_i(\omega) = 0$ for all $\omega \in \Omega$ for a given price function $P : \Omega \to \mathbb{R}_+$ that is $\Sigma(\Pi_i)$-measurable for all i. The resulting demand correspondence of agent i is then given as the $\Sigma(\Pi_i)$-measurable set-valued mapping $\varphi_i : \mathbb{R}^l_+ \times \Pi_i \rightrightarrows \Gamma_i$ such that for all $(p, I_i) \in \mathbb{R}^l_+ \times \Pi_i$,

$$\varphi_i(p, I_i) = \arg \max_{\{\theta_i\in\Gamma_i \mid p\theta_i=0\}} E\left[u_i(\theta_i, \omega), \pi_i(\omega \mid I_i)\right] \tag{2}$$

whereby $p = P(\omega)$ for $\omega \in I_i$.

The next two sections present two alternative equilibrium concepts for such economies.

3 Rational expectations equilibria

The concept of a REE takes private information partitions Π_i^{PI}, $i \in \{1, ..., n\}$, as primitives and defines equilibria through the consistency condition that agents base their supply-demand decisions on their private information augmented with the common information revealed through equilibrium prices.

Definition. Rational Expectations Equilibria.

Fix some private information-belief structure $\langle \Pi^{PI}, \pi \rangle$. A REE with respect to $\langle \Pi^{PI}, \pi \rangle$, denoted $\left(P^{REE}, \Theta^{REE}\right)\langle \Pi^{PI}, \pi \rangle$, is any mapping

$$\left(P_1^{REE}, ..., P_l^{REE}; \Theta_1^{REE}, ..., \Theta_n^{REE}\right) : \Omega \to \mathbb{R}^l_+ \times \mathbb{R}^{nl} \tag{3}$$

such that, for all $i \in \{1, ..., n\}$ and for all $\omega \in \Omega$,

(i)

$$\Theta_i^{REE}(\omega) \in \varphi_i\left(P^{REE}(\omega), I_i^{PI}(\omega) \cap I\left(P^{REE}(\omega)\right)\right) \tag{4}$$

with $I_i^{PI}(\omega) \in \Pi_i^{PI}$, $I\left(P^{REE}(\omega)\right) = P^{REE}(\omega)^{-1}$ and

(ii)

$$\sum_{i=1}^{n} \Theta_i^{RE}(\omega) = 0. \tag{5}$$

The standard motivation for the REE concept interprets the equilibrium price function P^{REE} as a *forecast function* that is *self-fulfilling* (cf. Radner 1979). Suppose for the moment that every agent i has some forecast function, say $P_i^F : \Omega \to \mathbb{R}_+^l$, which can be informally interpreted as agent i's ex-ante prediction for the market prices that will emerge in any given state $\omega \in \Omega$. A REE stipulates that, for all i, $P_i^F = P^{REE}$, whereby P^{REE} is the ex-post price-function that clears markets in every state of the world as defined above. That is, in a REE all agents share the same forecast function whose price predictions turn out to be correct ex-post (i.e., self-fulfilling). With respect to the information that the agents use to determine their demand-supply decisions, a REE is thereby closed off by the consistency condition that

$$I_i\left(P_i^F(\omega)\right) = I\left(P^{REE}(\omega)\right) \tag{6}$$

whereby $I_i\left(P_i^F(\omega)\right)$ is the event—being the same for every agent—that collects all states of the world in which the market-price takes on value $P^{REE}(\omega)$.

The focus of the present paper is on *fully revealing* REE $\left(P^{REE}, \Theta^{REE}\right)\left\langle \Pi^{PI}, \pi \right\rangle$ which satisfy, for all $\omega \in \Omega$,

$$I\left(P^{REE}(\omega)\right) = \{\omega\}. \tag{7}$$

That is, a fully revealing REE in our sense is characterized by the condition that every agent learns in such equilibrium the true state of the world from the market prices.

Remark 1. The literature on REE typically considers a state space

$$\Omega = E \times S \tag{8}$$

such that $S = S_1 \times ... \times S_n$ with generic element

$$\omega \equiv (e, s_1, ..., s_n). \tag{9}$$

S_i is here interpreted as the space of *private signals* that may be received by agent $i \in \{1, ..., n\}$ whereas the set E collects all remaining payoff-relevant characteristics that are ex ante unobservable. Private information partitions are then defined as

$$\Pi_i^{PI} = \{\mathbf{s}_i \mid s_i \in S_i\} \tag{10}$$

with

$$\mathbf{s}_i \equiv E \times S_1 \times ... \times \{s_i\} \times ... \times S_n \subseteq \Omega. \tag{11}$$

Under this approach, a *fully revealing* REE $\left(P^{REE}, \Theta^{REE}\right) \left\langle \Pi^{PI}, \pi \right\rangle$ is characterized by a one-one relationship between the signal space S and the space of equilibrium price values $P^{REE}(\Omega)$, i.e., for all $(e, s_1, ..., s_n) \in E \times S$,

$$I\left(P^{REE}(e, s_1, ..., s_n)\right) = E \times \{(s_1, ..., s_n)\}. \tag{12}$$

Formally, our definition (7) is the special case of (12) where we abstract—for the sake of analytical clarity—from the existence of any other payoff-relevant characteristics, i.e., we simply set $E = \{e\}$.

Remark 2. The literature on the existence of rational expectations equilibria investigates the question of whether there exists some REE $\left(P^{REE}, \Theta^{REE}\right) \left\langle \Pi^{PI}, \pi \right\rangle$ for any given private information partitions Π_i^{PI} and beliefs π_i, $i = \{1, .., n\}$. Kreps (1977) constructs the first example of non-existence of a REE whereby he observes that small perturbations of the parameter-values will re-establish existence. Subsequent results (Radner 1979, Allen 1981) establish generic existence of REE for economies that satisfy rather general regularity conditions such as convex asset-position spaces and expected utility functions that are strictly concave in asset-positions. More specifically, Radner (1979) proves for such economies that there exist fully revealing REE except for a Lebesgue measure zero subset of the parameter-space.

4 Rationalizable information equilibria

In a fully revealing REE, agent i's demand-supply decision in state ω lies in

$$\varphi_i\left(P^{REE}(\omega), I_i^{PI}(\omega) \cap I\left(P^{REE}(\omega)\right)\right) = \varphi_i\left(P^{REE}(\omega), \{\omega\}\right) \tag{13}$$

whereby the equilibrium price function P^{REE} works as a central coordination device for all agents. Consider now instead the decentralized decision situation in which highly sophisticated agents do not possess a coordination device but correctly understand the other agents' utility functions, private information partitions, and beliefs. Such highly sophisticated agents will be able to deduce information from the demand-supply decisions of the other agents at given market prices. For instance, suppose that agent j bases his demand-supply decision on the information

$$I_j = I_j^{PI} \cap I_j^* \tag{14}$$

where I_j^* denotes—still to be determined—all the information agent j infers from his observation of market variables. If a highly sophisticated agent i in our sense observes a specific realization of market variables, say (p, θ_j), such that θ_j maximizes j's utility only at information cell I_j, then agent i can deduce information I_j from his understanding of agent j's maximization problem.

The basic idea behind the concept of RIE is thus straightforward: If an agent observes the other agents' decisions, he can deduce information from this observation through his knowledge that the other agents have chosen a utility-maximizing decision. Our formal definition of a RIE closes this reasoning off by the consistency condition that (i) every agent is an utility maximizer, (ii) every agent knows that the other agents are utility maximizers, (iii) every agent knows (ii), and so forth. In short, we speak—in analogy to Bernheim (1984) and Pearce (1984)—of *rationalizable information* in the sense that it is common knowledge between all agents that they are utility maximizers with respect to their information and beliefs.[3]

Definition. Rationalizable information.

Fix some private information-belief structure $\left\langle \Pi^{PI}, \pi \right\rangle$ and some value $(p, \theta_1, ..., \theta_n)$ of observable market-variables. Initialize

$$I_i^0 (p, \theta_j) = \Omega \tag{15}$$

and define recursively, for all $k \geq 1$,

$$I_i^k (p, \theta_{-i}) = \bigcap_{j \neq i} I_i^k (p, \theta_j) \tag{16}$$

such that

$$I_i^k (p, \theta_j) = \left\{ \omega \mid \theta_j \in \varphi_j \left(p, I_j^{PI} (\omega) \cap I_j^{k-1} (p, \theta_{-j}) \right) \right\}. \tag{17}$$

Finally, define *agent i's rationalizable information at* $(p, \theta_1, ..., \theta_n)$ as

$$I_i^* (p, \theta_{-i}) \equiv \cap_{k=0}^{\infty} I_i^k (p, \theta_{-i}). \tag{18}$$

[3]While Pearce (1984, p. 1032) already draws an explicit connection between common knowledge (Aumann 1976) of the agents' maximization problem, on the one hand, and the formal definition of rationalizable strategy profiles, on the other hand, Brandenburger and Dekel (1987) as well as Tan and Werlang (1988) investigate in more detail in which sense such common knowledge serves as epistemic foundation of rationalizability.

The formal definition of a RIE requires that, in every state of the world, the value of market variables must be consistent with the agents' utility maximizing decisions with respect to the rationalizable information they can infer from these values.

Definition. Rationalizable Information Equilibria.

Fix some private information-belief structure $\left\langle \Pi^{PI}, \pi \right\rangle$. A RIE with respect to $\left\langle \Pi^{PI}, \pi \right\rangle$, denoted $\left(P^{RIE}, \Theta^{RIE}\right) \left\langle \Pi^{PI}, \pi \right\rangle$, is any mapping

$$\left(P_1^{RIE}, ..., P_l^{RIE}; \Theta_1^{RIE}, ..., \Theta_n^{RIE}\right) : \Omega \to \mathbb{R}_+^l \times \mathbb{R}^{nl} \tag{19}$$

such that, for all $i \in \{1, ..., n\}$ and for all $\omega \in \Omega$,

(i)

$$\Theta_i^{RIE}(\omega) \in \varphi_i \left(P^{RIE}(\omega), I_i^{PI}(\omega) \cap I_i^* \left(P^{RIE}(\omega), \Theta_{-i}^{RIE}(\omega)\right)\right) \tag{20}$$

with $I_i^*(\cdot, \cdot)$ defined by (18), and

(ii)

$$\sum_{i=1}^{n} \Theta_i^{RIE}(\omega) = 0. \tag{21}$$

Analogously to fully revealing REE, I will speak of a *fully revealing* RIE $\left(P^{RIE}, \Theta^{RIE}\right) \left\langle \Pi^{PI}, \pi \right\rangle$ iff every agent learns at equilibrium values the true state of the world, i.e., iff for all $i \in \{1, ..., n\}$ and all $\omega \in \Omega$,

$$I_i^* \left(P^{RIE}(\omega), \Theta_{-i}^{RIE}(\omega)\right) = \{\omega\}. \tag{22}$$

According to the following proposition, every fully revealing RIE can be interpreted as a fully revealing REE whereas the converse is not true.

Proposition.

(i) If there exists a fully revealing RIE $\left(P^{RIE}, \Theta^{RIE}\right) \left\langle \Pi^{PI}, \pi \right\rangle$, then there always exists a fully revealing REE $\left(P^{REE}, \Theta^{REE}\right) \left\langle \Pi^{PI}, \pi \right\rangle$ which is equivalent in the sense that

$$\begin{aligned} P^{REE} &= P^{RIE}, \\ \Theta^{REE} &= \Theta^{RIE}. \end{aligned} \tag{23}$$

(ii) If there exists a fully revealing REE $\left(P^{REE}, \Theta^{REE}\right)\left\langle \Pi^{PI}, \pi \right\rangle$, then there may not exist any equivalent RIE $\left(P^{RIE}, \Theta^{RIE}\right)\left\langle \Pi^{PI}, \pi \right\rangle$.

Proof.

Ad (i). This part is trivial and follows from construction. Let $P^{REE} = P^{RIE}$ and observe that, for all $i \in \{1, ..., n\}$ and all $\omega \in \Omega$,

$$\begin{aligned} \Theta_i^{REE}(\omega) &\in \varphi_i\left(P^{REE}(\omega), I_i^{PI}(\omega) \cap I\left(P^{REE}(\omega)\right)\right) \\ &= \varphi_i\left(P^{REE}(\omega), \{\omega\}\right) \\ &= \varphi_i\left(P^{RIE}(\omega), I_i^{PI}(\omega) \cap I_i^*\left(P^{RIE}(\omega), \Theta_{-i}^{RIE}(\omega)\right)\right) \end{aligned} \quad (24)$$

so that we can set $\Theta_i^{REE}(\omega) = \Theta_i^{RIE}(\omega)$, which proves the statement. Observe that the last line of the above argument follows from our assumption that the RIE is fully revealing.□

Ad (ii). Consider now a fully revealing REE $\left(P^{REE}, \Theta^{REE}\right)\left\langle \Pi^{PI}, \pi \right\rangle$.

Step 1. Assume for the moment that there exists some i and some $\omega \in \Omega$ such that

$$I_i^{PI}(\omega) \cap I_i^*(p, \theta_{-i}) \neq \{\omega\} \quad (25)$$

for $P^{REE}(\omega) = p$ and $\Theta_{-i}^{REE}(\omega) = \theta_{-i}$. In that case we can construct an economy for some $\left\langle \Pi^{PI}, \pi \right\rangle$ such that[4]

$$\begin{aligned} \varphi_i\left(P^{REE}(\omega), I_i^{PI}(\omega) \cap I\left(P^{REE}(\omega)\right)\right) &= \varphi_i(p, \{\omega\}) \\ &\neq \varphi_i\left(p, I_i^{PI}(\omega) \cap I_i^*(p, \theta_{-i})\right) \end{aligned} \quad (26)$$

to the effect that

$$\Theta_i^{REE}(\omega) \in \varphi_i\left(P^{REE}(\omega), I_i^{PI}(\omega) \cap I\left(P^{REE}(\omega)\right)\right) \quad (27)$$

but

$$\Theta_i^{REE}(\omega) \notin \varphi_i\left(p, I_i^{PI}(\omega) \cap I_i^*(p, \theta_{-i})\right). \quad (28)$$

In order to prove statement (ii), it is therefore sufficient to establish the possibility of situations satisfying (25).

Step 2. Let us thus investigate under which conditions (25) is satisfied. Suppose that for all θ_j in θ_{-i}

$$\theta_j \in \varphi_j\left(p, I_j^{PI}(\omega') \cap I_j^*(p, \theta_{-i})\right) \quad (29)$$

[4]We simply have to consider some belief π_i that attaches a sufficiently large probability to some state $\omega' \in I_i^{PI}(\omega) \cap I_i^*(p, \theta_{-i})$ such that $\omega' \neq \omega$ in order to obtain—for appropriately chosen utility functions—different utility-maximizing supply-demand decisions in the REE and the RIE case, respectively.

for some $\omega' \neq \omega$. In that case,

$$I_i^* (p, \theta_{-i}) = \{\omega, \omega', ...\} \tag{30}$$

so that (25) holds whenever Π^{PI} satisfies $\{\omega, \omega'\} \subseteq I_i^{PI}(\omega)$. As a consequence, statement (ii) can be proved by constructing situations such that for a given state ω with $P^{REE}(\omega) = p$ and $\Theta_{-i}^{REE}(\omega) = \theta_{-i}$ there exists another state ω' such that θ_{-i} is a maximizer for $j \neq i$ at price p with respect to the information available in ω'. That is, in such situations the optimal demand-supply decision of j is identical across different states. It is not difficult to construct such situations and in the following section I present an example that illustrates that such situations might even be quite relevant.□□

By the proof of Proposition (ii), we can immediately identify one condition that must necessarily hold whenever it is not possible to reinterpret a given REE as a RIE.

Corollary. Suppose that some fully revealing REE $\left(P^{REE}, \Theta^{REE}\right) \langle \Pi^{PI}, \pi \rangle$ cannot be reinterpreted as some RIE. Then there must be some agent i who has the same optimal demand-supply decision at different states of the world given his rationalizable information at REE values, i.e., there are $\omega, \omega' \in \Omega$ with $\omega \neq \omega'$ such that, for some $\theta_i \in \Gamma_i$,

$$\theta_i \in \varphi_i \left(p, I_i^{PI}(\omega) \cap I_i^* (p, \theta_{-i})\right) \tag{31}$$

as well as

$$\theta_i \in \varphi_i \left(p, I_i^{PI}(\omega') \cap I_i^* (p, \theta_{-i})\right) \tag{32}$$

with $p = P^{REE}(\omega)$ and $\theta_{-i} = \Theta_{-i}^{REE}(\omega)$.

To see the intuition behind the Corollary observe that the information-partition of an agent who augments his private information with his rationalizable information is always as least as coarse as the information partition of an agent who augments his private information with the information of a fully revealing forecast function (which is the finest partition possible in our set-up). In case both information partitions coincide for some REE price function P^{REE}, the agent's utility maximization problem under the RIE concept is the same as under the REE concept. If this holds for all agents, an according REE can always be reinterpreted as a RIE. However, in case the rationalizable information of an agent $j \neq i$ from observing (p, θ_i) is consistent with two different states ω and ω', this agent will have a strictly coarser information partition than in a fully revealing REE with $P^{REE}(\omega) = p$ and $\Theta_{-i}^{REE}(\omega) = \theta_{-i}$. Consequently, the optimal demand-supply decision of agent j under the RIE concept may be different than under the REE concept to the effect that such REE may not be recovered as a RIE.

5 Illustrative example 1: "The insider problem"

We illustrate the intuition behind the formal findings of the Proposition (ii) by a simple example that describes the most basic situation of a risky asset market with asymmetric information. Consider the state space

$$\Omega = \{\omega_1, \omega_2\} \tag{33}$$

and suppose that the asset is characterized by the payoff structure

$$X(\omega) = \begin{cases} 2 & \text{if } \omega = \omega_1 \\ 1 & \text{if } \omega = \omega_2 \end{cases} \tag{34}$$

Further suppose that there are two agents, 1 and 2, with private information partitions given by

$$\begin{aligned} \Pi_1^{PI} &= \{\{\omega_1\}, \{\omega_2\}\}, \\ \Pi_2^{PI} &= \{\Omega\}, \end{aligned} \tag{35}$$

respectively. Agent 1 is thus the "insider" who has perfect knowledge about the asset's performance, whereas agent 2 cannot directly observe the asset's performance. Assume that agent $i \in \{1, 2\}$ chooses at every information-cell $I_i \in \Pi_i$ some net-trade θ_i in $\{-1, 0, 1\}$ and that the demand-correspondence of agent i evaluated at information cell I_i and price p is given as

$$\varphi_i(p, I_i) = \arg \max_{\theta_i \in \{-1,0,1\}} E\left[u_i\left((X(\omega) - p) \cdot \theta_i\right), \pi_i(\omega \mid I_i)\right] \tag{36}$$

for some strictly increasing and strictly concave vNM utility functions u_i.

5.1 Rational expectations equilibria

It can easily be verified that there exist three different REE, denoted (a), (b), and (c), which are furthermore fully revealing. Namely, we have as equilibrium price function for all three equilibria

$$P^{REE}(\omega) = \begin{cases} 2 & \text{if } \omega = \omega_1 \\ 1 & \text{if } \omega = \omega_2 \end{cases} \tag{37}$$

so that—in accordance with no-trade theorems (see, e.g., Proposition 1 in Tirole 1982)—there do not exist any gains from trade in this example. The respective equilibrium allocations are given as

$$\Theta_1^{REE(a)}(\omega) = -\Theta_2^{REE(a)}(\omega) = \begin{cases} 0 & \text{if } \omega = \omega_1 \\ 0 & \text{if } \omega = \omega_2 \end{cases} \tag{38}$$

$$\Theta_1^{REE(b)}(\omega) = -\Theta_2^{REE(b)}(\omega) = \begin{cases} 1 & \text{if } \omega = \omega_1 \\ -1 & \text{if } \omega = \omega_2 \end{cases} \tag{39}$$

$$\Theta_1^{REE(c)}(\omega) = -\Theta_2^{REE(c)}(\omega) = \begin{cases} -1 & \text{if } \omega = \omega_1 \\ 1 & \text{if } \omega = \omega_2 \end{cases} \tag{40}$$

In words: REE (a) is the no-trade equilibrium; in REE (b) the informed agent 1 sells the asset in the bad and buys it in the good state; conversely, in REE (c) the informed agent 1 sells the asset in the good and buys it in the bad state.

According to REE (c) the fully revealing equilibrium price function P^{REE} guarantees the uninformed agent 2 that he is in the good state ω_1 whenever the informed agent 1 wants to sell the asset at price $p = 2$. This "guarantee", however, only comes from the defining REE assumption that no agent will be ex-post disappointed over his demand-supply decision. Since there is no actual mechanism in this model to save agent 2 from such ex-post disappointment, REE (c) appears to be a highly implausible equilibrium. In what follows I demonstrate that the RIE concept successfully excludes REE (c) as a possible equilibrium.

5.2 Rationalizable information equilibria

According to our definition of rationalizable information the uninformed agent 2 can infer after one iteration step, i.e., $k = 1$, the following information

$$I_2^1(p, \theta_1) = \left\{\omega \mid \theta_1 \in \varphi_1\left(p, I_1^{PI}(\omega) \cap \Omega\right)\right\} \tag{41}$$

from observing the market values (p, θ_1). Since the informed agent 1's maximization problem is solved by

$$\varphi_1(p, \{\omega_1\}) = \begin{cases} \{1\} & \text{if } p < 2 \\ \{-1, 0, 1\} & \text{if } p = 2 \\ \{-1\} & \text{if } p > 2 \end{cases} \tag{42}$$

and

$$\varphi_1(p, \{\omega_2\}) = \begin{cases} \{1\} & \text{if } p < 1 \\ \{-1, 0, 1\} & \text{if } p = 1 \\ \{-1\} & \text{if } p > 1, \end{cases} \tag{43}$$

(41) becomes in this example

$$I_2^1(p,\theta_1) = \begin{cases} \{\omega_2\} & \text{if } \theta_1 = -1 \text{ and } 1 \leq p < 2 \\ \{\omega_1\} & \text{if } \theta_1 = 1 \text{ and } 1 < p \leq 2 \\ \{\omega_2\} & \text{if } \theta_1 = 0 \text{ and } p = 1 \\ \{\omega_1\} & \text{if } \theta_1 = 0 \text{ and } p = 2 \\ \Omega & \text{if } \theta_1 = -1 \text{ and } 2 \leq p \\ \Omega & \text{if } \theta_1 = 1 \text{ and } p \leq 1 \\ \emptyset^5 & \text{else.} \end{cases} \tag{44}$$

Furthermore, the iterative process already terminates after one step implying

$$I_2^*(p,\theta_1) = I_2^1(p,\theta_1) \tag{45}$$

as well as

$$I_2^{PI}(\omega) \cap I_2^*(p,\theta_1) = I_2^*(p,\theta_1) \text{ for all } \omega \in \Omega. \tag{46}$$

As in the case of REE it is not difficult to see that any RIE price function P^{RIE} must satisfy

$$P^{RIE}(\omega) = \begin{cases} 2 & \text{if } \omega = \omega_1 \\ 1 & \text{if } \omega = \omega_2 \end{cases} \tag{47}$$

Furthermore, it is also easy to verify that the REE (a) and (b) of the previous subsection are also RIE. For example, consider the REE (b) and observe that

$$\begin{aligned} & \varphi_2\left(P^{RIE}(\omega_1), I_2^{PI}(\omega_1) \cap I_2^*\left(P^{RIE}(\omega_1), \Theta_1^{REE(b)}(\omega_1)\right)\right) \\ = \; & \varphi_2\left(p = 2, I_2^1(p = 2, \theta_1 = 1)\right) \\ = \; & \varphi_2(p = 2, \{\omega_1\}) \\ = \; & \{-1\} \ni \Theta_2^{REE(b)}(\omega_1) \end{aligned} \tag{48}$$

as well as

$$\begin{aligned} & \varphi_2\left(P^{RIE}(\omega_2), I_2^{PI}(\omega_2) \cap I_2^*\left(P^{RIE}(\omega_2), \Theta_1^{REE(b)}(\omega_2)\right)\right) \\ = \; & \varphi_2\left(p = 1, I_2^1(p = 1, \theta_1 = -1)\right) \\ = \; & \varphi_2(p = 1, \{\omega_2\}) \\ = \; & \{1\} \ni \Theta_2^{REE(b)}(\omega_2). \end{aligned} \tag{49}$$

That is,

$$P^{REE} = P^{RIE} \text{ and } \Theta^{REE(b)} = \Theta^{RIE(b)}. \tag{50}$$

On the other hand, however, there does not correspond any RIE to the REE (c). To see this let $P^{RIE} = P^{REE}$ and $\Theta_1^{RIE(c)} = \Theta_1^{REE(c)}$ and observe that

$$\begin{aligned} & \varphi_2\left(P^{RIE}(\omega_1), I_2^{PI}(\omega_1) \cap I_2^*\left(P^{RIE}(\omega_1), \Theta_1^{REE(c)}(\omega_1)\right)\right) \\ = & \varphi_2\left(p=2, I_2^1(p=2, \theta_1=-1)\right) \\ = & \varphi_2(p=2, \Omega) \\ = & \{-1\} \end{aligned} \tag{51}$$

whereby the last step follows from the assumption that π_2 has full support on Ω.[6] That is, if agent 2 observes price $p = 2$ and $\theta_1 = -1$, he remains uninformed about the true state of the world implying

$$\Theta_2^{REE(c)}(\omega_1) = 1 \notin \{-1\} = \varphi_2\left(p=2, I_2^*(p=2, \theta_1=-1)\right). \tag{52}$$

As a consequence, the markets do not clear in state ω_1 so that the REE (c) cannot be reinterpreted as a RIE.

In order to see the economic rationale behind the finding that REE (c) is not a RIE observe that REE (c) corresponds to a situation in which (29) is satisfied since

$$\begin{aligned} \{-1\} & \in \varphi_1(p=2, \{\omega_1\}) \\ & \text{as well as} \\ \{-1\} & \in \varphi_1(p=2, \{\omega_2\}) \end{aligned} \tag{53}$$

to the effect that

$$I_2^*(p=2, \theta_1=-1) = \Omega. \tag{54}$$

In words: At price $p = 2$, the informed agent 1 would sell the asset in both states, i.e., in the good as well as in the bad state, so that the highly sophisticated—but uninformed—agent 2 cannot infer from the observation $(p = 2, \theta_1 = -1)$ the true state of the world. The unique utility maximizing choice of agent 2 in this situation is to also offer the asset at price $p = 2$ implying a violation of the market clearing condition of a RIE.

6 Illustrative example 2: "The lemons market problem"

Mas-Colell, Whinston, and Green (1995) already observe that Radner's (1979) original definition of a REE does not refer to any information that may be deduced from an agent's knowledge about the utility-maximizing behavior of other agents. In their words:

[6] Selling, i.e., $\theta_2 = -1$, at price $p = 2$ results in a payoff distribution for agent 2 that first-order stochastically dominates the according distributions from buying, i.e., $\theta_2 = 1$, or doing nothing, i.e., $\theta_2 = 0$, whereby this domination becomes strict under our full-support assumption.

> "Up to now, prices may have conveyed information about an exogenously occurring state. But in a world of asymmetric information, prices could also convey information on the consumers' endogenously chosen actions, and those actions could matter for individual utilities. For example, the final utility of a consumer may depend not only of the number of units consumed and on exogenous states but also on some other statistic depending on other consumers' actions. If we regard this statistic as a "state" then it is as if states were determined endogenously." (p. 724, Mas-Colell et al., 1995)

Unfortunately, Mas-Colell et al. (1995) do not offer any general theory about the introduction of "endogenous" state spaces in order to incorporate the idea that the agents' actions may reveal information. Rather, they consider a specific example (i.e., "The lemons market", Example 19.H.6.) for which they informally argue how an agent should utilize his knowledge about the utility-maximizing behavior of the other agents. With "informal" I mean that Mas-Colell et al. just write "A potential buyer will deduce..." (p. 724, ibid.) without offering a general formal framework that describes (i) how the agents make their deductions and (ii) how these deductions enter into the updating of probability measures defined for an appropriately chosen event-space. In the previous section we have already seen how the RIE concept captures the idea that agents' actions may reveal information when the state space Ω contains "exogenous", i.e., demand-supply decision independent, states. In the remainder of this section, I reformulate a slightly modified version[7] of the "lemons market" example in order to demonstrate that the RIE concept is also the appropriate formal framework for recapturing—and generalizing—the informal argumentation of Mas-Colell et al. (1995) for "endogenous" state spaces Ω.

Suppose that there are three agents: The potential buyer B; a potential high-type seller SH, offering a high quality commodity; and a potential low-type seller SL, offering a low quality commodity (i.e., the "lemon"). Whereas the low-type seller has a strict incentive to supply his commodity at any positive market price, the high-type seller's decision to supply or not supply will depend on whether the market price (weakly) exceeds his evaluation of 1 for the high-quality commodity. When both potential sellers decide to supply the commodity, only one of them will have the opportunity to sell it on the market to the potential buyer.[8] We assume that in this case both potential sellers have an equal chance to sell their commodity to the seller. While the potential buyer

[7] Mas-Colell et al. (1995) consider a relaxed market clearing condition: "It is natural to say that if at the admissible pair [...] the total demand is not larger than the total supply then we are at a *rational expectations* equilibrium." (p. 724, ibid.). Since this gives rise to—in my opinion—implausible equilibria, I have reformulated the example to the effect that markets clear iff supply equals demand.

[8] We may think of this situation as a "first come to the market, first sell" situation.

cannot differentiate between the supply decision of the high-type seller, denoted θ_{SH}, and the supply decision of the low-type seller, denoted θ_{SL}, he can observe the total market supply

$$|\theta_S| = |\min\{\theta_{SH}, \theta_{SL}\}|. \tag{55}$$

Consider now the state space

$$\Omega = \{\omega_1, \omega_2\} \tag{56}$$

and suppose that the demand-correspondences of the two potential sellers are given as

$$\varphi_{SH}(p, \{\omega_1\}) = \begin{cases} \{-1\} & \text{if } p \geq 1 \\ \emptyset & \text{else} \end{cases} \tag{57}$$

$$\varphi_{SH}(p, \{\omega_2\}) = \begin{cases} \{0\} & \text{if } p < 1 \\ \emptyset & \text{else} \end{cases} \tag{58}$$

and

$$\varphi_{SL}(p, \Omega) = \begin{cases} \{-1\} & \text{if } p > 0 \\ \{0\} & \text{if } p = 0 \end{cases}, \tag{59}$$

whereby we assume that the sellers have the following information partitions

$$\begin{aligned} \Pi_{SH}^{PI} &= \{\{\omega_1\}, \{\omega_2\}\}, \\ \Pi_{SL}^{PI} &= \{\Omega\}. \end{aligned} \tag{60}$$

That is, through (57) we associate the "endogenous" state ω_1 with the high-type seller's utility maximizing decision to supply his commodity. Similarly, through (58) we associate the "endogenous" state ω_2 with the high-type seller's optimal decision not to supply his commodity. By (59), the low-type seller would like to sell his commodity at any price $p > 0$.

Assume that the potential buyer's private information partition is given as

$$\Pi_B^{PI} = \{\Omega\} \tag{61}$$

and that he receives the following payoff from purchasing the commodity

$$X_B(\omega) = \begin{cases} \frac{1}{2} \cdot x(high) + \frac{1}{2} \cdot x(low) & \text{if } \omega = \omega_1 \\ x(low) & \text{if } \omega = \omega_2 \end{cases} \tag{62}$$

such that

$$\begin{aligned} x(high) &= 4, \\ x(low) &= 0. \end{aligned} \tag{63}$$

The interpretation of (62) is that the risk-neutral buyer has in state ω_1 an equal chance to either purchase the high or the low quality commodity whereas he would purchase in state ω_2 the low quality commodity with certainty. The potential buyer's demand-correspondence at price p and information cell I_B is given as

$$\varphi_B(p, I_B) = \arg\max_{\theta_B \in \{0,1\}} E\left[(X_B(\omega) - p) \cdot \theta_B, \pi_B(\omega \mid I_B)\right] \tag{64}$$

with $\pi_B(\omega_1), \pi_B(\omega_2) > 0$.

In order to solve this economy for RIE observe at first that the market clearing condition (21) becomes, by (55),

$$\Theta_B^{RIE}(\omega) + \min\left\{\Theta_{SH}^{RIE}(\omega), \Theta_{SL}^{RIE}(\omega)\right\} = 0 \text{ for all } \omega. \tag{65}$$

According to our concept of rationalizable information, the potential buyer infers after one iteration step, i.e., $k = 1$, the information

$$I_B^1(p, \theta_S) = \left\{\omega \mid \theta_{SH} \in \varphi_{SH}\left(p, I_{SH}^{PI}(\omega)\right), \theta_{SL} \in \varphi_{SL}(p, \Omega)\right\} \tag{66}$$

from observing the market values (p, θ_S) whereby this iterative process terminates after one step, i.e.,

$$I_B^*(p, \theta_S) = I_B^1(p, \theta_S). \tag{67}$$

In particular, we have

$$I_B^*(p, \theta_S) = \begin{cases} \{\omega_1\} & \text{if } p \geq 1 \\ \{\omega_2\} & \text{if } p < 1, \end{cases} \tag{68}$$

implying for the potential buyer's demand-correspondence (64) at his rationalizable information

$$\varphi_B(p, I_B^*(p, \theta_S)) = \begin{cases} \{0\} & \text{if } p > 2 \\ \{0,1\} & \text{if } p = 2 \\ \{1\} & \text{if } 1 \leq p < 2 \\ \{0\} & \text{if } 1 < p < 2 \\ \{0,1\} & \text{if } p = 0. \end{cases} \tag{69}$$

Consequently, the possible RIE, $\left(P^{RIE}, \Theta_B^{RIE}, \Theta_{SH}^{RIE}, \Theta_{SH}^{RIE}\right)$, of this economy are characterized—under the market clearing condition (65)—by the equilibrium allocation

$$\left(\Theta_B^{RIE}, \Theta_{SH}^{RIE}, \Theta_{SL}^{RIE}\right)(\omega) = \begin{cases} (1, -1, -1) & \text{if } \omega = \omega_1 \\ (0, 0, 0) & \text{if } \omega = \omega_2 \end{cases} \tag{70}$$

supported by any equilibrium price function satisfying the following conditions

$$\begin{aligned} 1 &\leq P^{RIE}(\omega_1) \leq 2 \\ 0 &= P^{RIE}(\omega_2). \end{aligned} \tag{71}$$

Observe that the states ω_1 and ω_2 are "endogenous" in the sense of Mas-Colell et al. (1995) and that we may interpret these different "endogenous" states as different Walrasian equilibria: In state ω_1 the Walrasian auctioneer announces some price p such that $1 \leq p \leq 2$, which clears the market through trade; in state ω_2 the Walrasian auctioneer announces the price $p = 0$, which clears the market through no-trade. The RIE concept offers a "solution" for this endogenous state space set-up in terms of an equilibrium price function that assigns to the endogenous state ω_1 the Walrasian trade equilibrium whereas it assigns to the endogenous state ω_2 the Walrasian no-trade equilibrium. The fact that the RIE concept cannot pin down any 'objective' probabilities for these endogenous states corresponds to the fact that the Walrasian equilibrium concept cannot pin down any probabilities for different equilibria if there are multiple Walrasian equilibria. The potential buyer's beliefs $\pi_B(\omega_1), \pi_B(\omega_2) > 0$ are therefore purely subjective in this endogenous state space set-up.

Finally, notice that we may reinterpret, by Proposition (ii), the fully-revealing RIE of this economy as fully revealing REE. By construction, the high-type seller has in any REE different optimal supply-demand decisions in different states of the world. Using the argumentation of the Corollary, it can easily be shown that the both sets of equilibria—RIE, on the one hand; REE, on the other hand—coincide for this example.

7 Relationship to the rational expectations literature

Recall that the notion of *rational expectations*, in terms of the *rational expectations hypothesis*, goes back to Muth (1961) who speaks of rational expectations whenever agents have an unbiased point-estimator of the economy's price process. More precisely, Muth considers an economy which is characterized by (linear) aggregate supply- and demand functions—standing in for the production- and the consumption side, respectively, of the economy—whereby the aggregate supply is subject to random shocks.[9] According to Muth's rational expectation hypothesis, all producers base their utility maximizing ex-ante supply-production decision on an expected price that is the unbiased estimator for the ex-post market-clearing equilibrium price. Whereas Radner's REE concept is a general equilibrium concept that applies to any economy under asymmetric information, Muth's economic framework of linear aggregate supply- and demand functions (whereby supply decisions depend on expected prices rather than on price distributions) is only

[9] Later generalizations of this linear macroeconomic set-up (e.g., DeCanio 1979) also allow for random shocks on the demand-side.

valid under very specific microeconomic conditions. More importantly, under Radner's rational expectations concept the agents' common forecast function must coincide with the economy's equilibrium price function in (almost) all states of the world whereby the agents' subjective probability measures do not necessarily have to coincide with any 'objective' probability measure. That is, Radner's agents completely understand how prices emerge in any possible state of the world without necessarily sharing a common 'objective' probability distribution over this price function. In contrast, Muth requires his rational expectations agents to have a point-forecast—instead of a whole forecast function—which must coincide with the expected value of the economy's equilibrium price function with respect to some 'objective' probability measure.

Muth's (1961) rational expectations hypothesis has sparked a rich literature that investigates conditions under which agents with subjective probability measures may learn (and may actually have economic incentives to learn) an unbiased point-estimator for the economy's supposedly objective price process; (DeCanio 1979, Evans 1985, for surveys see Grinspun 1995, and Evans and Honkapohja 2001, 2009). Within this literature on the possible coincidence of subjective estimators and the mean of an objective price-process, there exists a game-theoretic literature that investigates a possible justification of Muth's rational expectations hypothesis through game-theoretic solution concepts. In particular, Guesnerie (1992), Evans and Guesnerie (1993, 2005), Heinemann (1997) and Guesnerie and Jara-Moroni (2009) characterize economic situations for which Muth's rational expectations hypothesis can be justified through an application of the rationalizability concepts of Bernheim (1984), Moulin (1984), and Pearce (1984). For example, Guesnerie (1992) reinterprets a deterministic Muth-like economy as a strategic game of Cournot-output competition between producers to the effect that game-theoretic solution concepts such as Nash equilibrium and (point) rationalizability become applicable. Guesnerie (1992) then speaks of "rationalizable expectations equilibria" whenever the corresponding supply-production decisions are supported by some (point) rationalizable strategy profile of the corresponding Cournot game.[10] In contrast to the deterministic economy in Guesnerie (1992)—in which all uncertainty about the price-process refers to "strategic uncertainty" (i.e., a player's uncertainty about the other players' strategy choice)—Evans and Guesnerie (1993) describe a game of Cournot output competition between producers who face—in addition to "strategic uncertainty"—a random aggregate demand function.

[10] Guesnerie (1992) also speaks of "strong rational expectations equilibria" whenever the corresponding strategic solution is very robust in the sense that it is given as the unique (point) rationalizable strategy profile, i.e., whenever there exists a unique Nash equilibrium in the corresponding Cournot-game that happens to be also the unique (point)-rationalizable strategy profile.

It is important to emphasize that—despite the reference to *rationalizable information*—the RIE concept is not a game-theoretic solution concept for some strategic reformulation of exchange economies under asymmetric information in the sense of, e.g., Guesnerie (1992) or Heinemann (1997), who use rationalizable strategy profiles as a motivation for specific competitive equilibria. Rather, a RIE is defined as a general equilibrium concept under asymmetric information with the intention to determine an equilibrium through epistemic foundations that go beyond Radner's (1979) definition of a REE in terms of a mere self-fulfilling forecast function. The concept of a RIE thereby adopts from game-theory the epistemic reasoning of rationalizability and applies it as consistency condition to a general equilibrium context. More specifically, a RIE is a general equilibrium concept which is characterized by a price- and allocation function such that the values of this price- and allocation function are in every state of the world consistent with the assumption that the agents' respective expected utility maximization problems are common knowledge between the agents.

8 Concluding remarks

The formal definition of an *rational expectations equilibrium* (=REE) is characterized by the consistency condition that the equilibrium price function is a self-fulfilling forecast function. The existing literature thereby treats this consistency condition as equivalent to the notion that all agents are "highly sophisticated" without assigning any explicit definition to this term. The present paper proposes a formal definition of highly sophisticated agents through the notion of "rationalizable information" according to which agents infer information from observed market-variables through their common knowledge that all agents are expected utility maximizers. The corresponding concept of *rationalizable information equilibria* (=RIE) is characterized by the consistency condition that the equilibrium price function clears markets in every state of the world whereby the agents' demand-supply decisions are consistent with their rationalizable information.

The following main findings emerge from our analysis:

1. While the assumption of sophisticated agents in our sense gives rise to RIE price functions that can always be interpreted as self-fulfilling forecast functions, the converse is not true. That is, not every self-fulfilling REE price function can be supported by the assumption of sophisticated agents in our sense.

2. The defining consistency condition of a fully revealing REE is equivalent to the institution of a Walrasian auctioneer who announces in every state of the world (i) the true state as well as (ii) the corresponding equilibrium price. That is, this

Walrasian auctioneer ensures that the true state of the world becomes common knowledge to all agents. In contrast, the defining consistency condition of a fully revealing RIE assumes that "only" the agents' utility maximization problems are common knowledge.

3. Consequently, the RIE concept is a plausible refinement of the REE concept whenever one believes that market information is processed by highly sophisticated agents in a decentralized manner rather than through the central coordination device of a state-announcing Walrasian auctioneer.

Finally, notice that our "insider problem" example shows that beyond the mere difference in their notions of sophisticated agents—i.e., the notion of a self-fulfilling forecast function, on the one hand; the notion of rationalizable information, on the other hand—the embrace of either one equilibrium concept would also have practical implications. Namely, both equilibrium concepts come up with different answers to the question of whether market trade between sophisticated agents with asymmetric information requires legal protection of uninformed investors or not: Whereas the REE concept allows for an equilibrium such that sophisticated but uninformed investors buy from insiders in the absence of any legal protection, such an equilibrium is not viable under the RIE concept for the considered example.

References

Allen, B. (1981) "Generic existence of completely revealing equilibria for economies with uncertainty when prices convey information," *Econometrica* **49**, 1173-1199.

Aumann, R.J. (1976) "Agreeing to disagree," *Annals of Statistics* **4**, 1236-1239.

Bernheim, B.D. (1984) "Rationalizable strategic behavior," *Econometrica* **52**, 1007-1028.

Brandenburger, A. and E. Dekel (1987) "Rationalizability and correlated equilibria," *Econometrica* **55**, 1391-1402.

DeCanio, S.J. (1979) "Rational expectations and learning from experience," *Quarterly Journal of Economics* **93**, 47-57.

Evans, G. (1985) "Expectational stability and the multiple equilibria problem in linear rational expectations models," *Quarterly Journal of Economics* **100**, 1217-1233.

Evans, G. W. and R. Guesnerie (1993) "Rationalizability, strong rationality and expectational stability," *Games and Economic Behavior* **5**, 632-646.

Evans, G. W. and R. Guesnerie (2005) "Coordination on saddle-path solutions: The eductive viewpoint—linear multivariate models," *Journal of Economic Theory* **124**, 202-229.

Evans G.W. and S. Honkapohja (2001) *Learning and expectations in macroeconomics.* Princeton University Press.

Evans, G.W. and S. Honkapohja (2009) "Expectations, learning and monetary policy: An overview of recent research," in *Monetary policy under uncertainty and learning*, ed. by Klaus Schmidt-Hebbel and Carl Walsh, Volume 13, Chapter 2. Central Bank of Chile.

Grinspun, R. (1995) "Learning rational expectations in an asset market," *Journal of Economics* **61**, 215-243.

Guesnerie, R. (1992) "An exploration on the eductive justifications of the rational expectations hypothesis," *American Economic Review* **82**, 1254-1278.

Guesnerie, R. and P. Jara-Moroni (2009) "Expectational coordination in simple economic contexts: Concepts and analysis with emphasis on strategic substitutabilities," Working paper no. 2009 - 27, Paris School of Economics.

Heinemann, F. (1997) "Rationalizable expectations and sunspot equilibria in an overlapping-generations economy," *Journal of Economics* **65**, 257-277.

Kreps, D. (1977) "A note on 'fulfilled expectations' equilibria," *Journal of Economic Theory* **14**, 32-43.

Mas-Colell, A., Whinston, M.D., and J.R. Green (1995) *Microeconomic Theory.* Oxford University Press.

Milgrom, R. and N. Stokey (1982) "Information, trade, and common knowledge," *Journal of Economic Theory* **26**, 17-27.

Moulin, H. (1984) "Dominance solvability and cournot stability," *Mathematical Social Sciences* **7**, 83-102.

Muth, J.F. (1961) "Rational expectations and the theory of price movements," *Econometrica* **29**, 315-335.

Pearce, D.G. (1984) "Rationalizable strategic behavior and the problem of perfection," *Econometrica* **52**, 1029-1050.

Radner, R. (1979) "Rational expectations equilibrium: Generic existence and the information revealed by prices," *Econometrica* **47**, 655-678.

Tan, T. and S.R. Costa Da Werlang (1988) "On Aumann's notion of common knowledge, an alternative approach," *Journal of Economic Theory* **45**, 370-391.

Tirole, J. (1982) "On the possibility of speculation under rational expectations," *Econometrica* **50**, 1163-1181.