The stationarity of financial time series

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Abstract

The concept of stationarity has always been central to econometric time series analysis, since most financial time series analysis necessitates that data be made stationary before any regressions can be performed. It has become common practice to transform non-stationary financial time series by either differencing data to the order *I(1)* or using log-normal returns. However, this process often leaves the data bereft of its descriptive value and therefore ineffective for financial time series analysis. This work challenges the common practice of differencing data indiscriminately by presenting an overview of what has been established through other methods of achieving stationarity, including fractional differencing and de-trending. The assumption that time series data are first difference stationary (and that the correct form of differencing should be performed before attempting any regression analysis or forecasting) is challenged.

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1. Introduction

In applied econometric analysis concerning time series data, the prerequisite of stationarity is a well-known concept. A stationary time series is represented by data over time whose statistical properties remain constant regardless of a change in the time origin (Fielitz, 1971:1025). In financial markets, a financial time series can be represented by historical stock prices. Research conducted on financial markets suggests that financial time series follow a random walk (Fama, 1970). A random walk process is inherently non-stationary because of the presence of a unit root (Burke & Hunter, 2005:22). When a time series contains a unit root it is necessary to difference the time series to render it stationary (Box & Jenkins, 1976). The first difference approach has also become popular mainly because many microeconomic time series are difference stationary and not trend stationary (Nelson and Plosers, 1982). It has become common practice to transform non-stationary financial time series by either differencing the data to the order *I(1)* or using the log-normal returns. Simple differencing could over-difference a time series (Burke *et al.*, 2005:23) leaving the data void of

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their descriptive value and therefore ineffective in financial time series analysis. This paper aims to challenge the common practice of differencing data indiscriminately by presenting an overview of what has been established through other methods of achieving stationarity. These methods include, but are not limited to, achieving stationarity through fractional differencing and de-trending.

In Section 2, this paper briefly provides an overview of the different forms of stationarity found in financial time series data. Section 3, provides an in-depth review of the literature surrounding these different forms of stationarity and Section 4 draws attention to the different tests used to determine the stationarity of financial time series (including unit root tests and tests with stationarity as the null-hypothesis). A conclusion and critical overview of the findings are given in Section 5.

2. A brief overview of the forms of stationarity in time series

The terms non-stationary and stationary form a fundamental part of time series econometric analysis. A stationary time series refers to data whose statistical properties remain unchanged over time regardless of the change in time (Fielitz, 1971:1025). Investigating time series data for the purpose of obtaining significant properties of these time series will be meaningless if the data are non-stationary or cannot be transformed to be stationary (Fielitz, 1971:1025). Using non-stationary time series in regression analysis can lead to spurious regression (Asteriou and Hall, 2007:293). Capturing and examining the properties of financial time series, therefore, requires that univariate financial time series are stationary before examining such data.

Stationarity can take several different forms. Firstly, a time series is considered strictly stationary if the data properties remain unaffected by a shift in the time origin (Maddala & Kim, 2000:10; Montgomery, Jennings & Kulahci, 2008:25). Secondly, a time series is covariant stationary when it exhibits the following characteristics:

i) the time series is mean reverting,[4] and

ii) a finite variance can be observed in the time series as the lag length increases when observing a correlogram of the time series. This theoretical correlogram diminishes

[4] The series will fluctuate around a constant, long-run mean.

faster than the theoretical correlogram of non-stationary time series (Asteriou et al., 2007:231).

Thirdly, data may be trend-stationary, indicating that they comprise of stationary variances around a linear trend and they are made stationary by removing this linear trend (Burke *et al.*, 2005:30; Enders, 2010:191). Fourthly, data are difference-stationary if they contain a unit root and can be rendered stationary by differencing them according to their level of integration (McCabe & Tremayne, 1995:1015; Laybourne, McCabe & Tremayne, 1996:435); Enders, 2010:192). Fifthly, a time series may be fractionally integrated and will require fractional differencing to reduce the time series to stationarity (Burke *et al.*, 2005:31), Finally, cyclical and seasonal components might be present in data and they might still be stationary after both first differencing and seasonally differencing them respectively (Enders, 2010:192 and Montgomery *et al.*, 2008:39). A time series may therefore also be either cyclical-stationary or seasonal-stationary.

Subsequent sections of this work will provide a critical overview of the different forms of stationarity that may be achieved in financial time series analysis by means of an extensive literature review.

3. Stationarity

3.1 Strictly stationary time series

The first type of stationarity to be reviewed is that of a strictly stationary process. According to Maddala *et al*. (2000:10) and Montgomery, Jennings and Kulahci (2008:25) a time series is strictly stationary if its properties remain unaffected by a shift in the time origin. A time series is, therefore, strictly stationary if the distribution of the series x_t, x_{t+1},..., x_{t+n} is equal to the joint distribution of the series x_{t+k}, x_{t+k+1},..., x_{t+k+n}. A strictly stationary time series is further characterised as having a constant mean and constant variance (Maddala *et al*., 2000:27).

3.2 Covariance stationary time series

A time series is defined as being covariance stationary (Enders, 2010:54) if it exhibits long memory and finite variance. Also, as the lag length increases when observing a correlogram of the time series, the theoretical correlogram diminishes faster than a theoretical correlogram of a non-stationary time series (Asteriou *et al*., 2007:231). Mathematically the characteristics of a covariance stationary time series can be expressed as follows (Enders, 2010:54):

$$E(x_t) = E(x_{t-s}) = \mu \quad (1)$$

$$E[(x_t - \mu)^2] = E[(x_{t-s} - \mu)^2] = \sigma_x^2 \quad (2)$$

$$E[(x_t - \mu)(x_{t-s} - \mu)] = E[(x_{t-j} - \mu)(x_{t-j-s} - \mu)] = \gamma_s \quad (3)$$

Where Equation 1 refers to the time series and constant mean, Equation 2 describes constant variance of the time series, and Equation 3 establishes that the time series has a constant covariance over time. The subscript *t* is the time period, *s* is the shift in the time origin and *j* represents the number of lags. Covariance stationary time series are also known to be weakly-stationary, second-order-stationary or wide-order stationary (Enders, 2010:54). A time series that does not exhibit these characteristics is thus non-stationary, by definition.

3.3 Differencing and stationarity

A time series is differenced stationary if the series contains a unit root and can be rendered stationary by differencing the time series according to the level of integration (McCabe *et al.*, 1995:1015, Laybourne *et al.,* 1996:435 and Enders, 2010:192). The level of integration can be an integer value or a non-integer value.

3.3.1 Differenced stationary financial time series

The first difference approach has become popular mainly because of the work of Nelson and Plosers (1982) who argued that many micro-economic time series are difference stationary and not trend stationary. The widespread popularity of differencing a time series to achieve stationarity may be attributed to McCabe *et al*. (1995:1015),[5] ii) Leybourne *et al*. (1996:45),[6] and iii) Burke *et al*. (2005:22).[7]

The first difference of a process refers to the change within the process from one time period to the next (Burke *et al*., 2005:22). The first difference of a time series x_t is therefore, $x_t - x_{t-1}$ and is denoted by Δx_t. According to Burke *et al*. (2005:22), $\Delta x_t = \varepsilon_t$ with ε_t being a

[5] "The cornerstone of practical time series modelling is their acceptability of the difference stationary assumption".
[6] "Much of modern applied econometric analysis is predicated on the assumption that data series concerned are non-stationary and that … they can be differenced to achieve stationarity".
[7] "... taking the first difference of the non-stationary process has reduced it to stationarity".

white noise process and Δx_t therefore being a stationary process. It is also possible to take the second difference of a time series, i.e $\Delta^2 x_t = \Delta\varepsilon_t$. But, if a time series that is *I(1)* is second differenced the time series will be over-differenced so care should be taken that a time series is only differenced by the minimal number of times needed to render the series stationary (Burke *et al.*, 2005:23).

Besides being integrated to the order of one or two and requiring first or second differencing to achieve stationarity, a time series may also be *fractionally* integrated. A fractionally integrated time series needs to be fractionally differenced in order to render the time series stationary.

3.3.2 Fractionally differenced stationary financial time series

First differencing is used by most economists as an alternative to fractional differencing due to the difficulties associated with the latter method (Erfani and Samimi, 2009:1721), but, by replacing fractional differencing with first differencing, the data are often over differenced and inherent properties of the data are lost. The importance of fractional differencing, therefore, lies in the fact that data may be reduced to stationarity without over differencing and therefore, retaining properties essential to forecast modelling accuracy.

The order of differencing used during the fractional differencing of the financial time series is determined by calculating the specific series' fractional differencing parameter, *d*. Several different methods of determining *d* have emerged in the empirical literature. Authors of these methods include Hurst (1951), Mandelbrot (1972:259), Davies and Hart (1987:95), Hosking (1981:175), Lo (1991) and Peng, Havlin, Stanley and Goldberger (1994:82). The *d*-parameter is derived from determining the Hurst coefficient (*H*) of a time series which indicates long memory properties of the time series (Garcia, Percival, Cannon, Raymond and Bassingwaighte, 1997:10). *H* may be estimated by using rescaled range analysis (R/S) and de-trended fluctuation analysis (DFA) (Caccia *et al.*, 1997:610 and Erfani *et al.*, 2009:172). This paper provides a discussion of the R/S method followed by the subsequent steps needed to determine *d*: the DFA method is discussed in Erfani *et al* (2009:1723). The Hurst coefficient was developed in 1951 by applying rescaled analysis (R/S statistic) to the presence of long-range correlations within time series (Lo, 1991:1280). The R/S statistic is determined by dividing time series into windows, calculating the mean of each window, determining the range of the cumulative sum of the observations within each window, and dividing by the corresponding windows standard deviation (Lo, 1991:1286; Caccia *et al.*, 1997:614; Erfani *et al.*, 2009:173). The R/S statistic is denoted by R/S_n and defined as:

$$R/S_n \equiv \frac{1}{s_n}\left[\underset{0\le k\le n}{Max}\sum_{j=1}^{k}(X_t - \bar{X}_n) - \underset{0\le k\le n}{Min}\sum_{j=1}^{k}(X_t - \bar{X}_n) \right] \tag{3}$$

where s_n is the standard deviation estimator:

$$s_n \equiv \sqrt{\sum_j \frac{(X_j - \bar{X}_n)^2}{n}} \tag{4}$$

Equations 3 and 4 are applied to each window and the *R/S* is obtained for all time periods (Erfani *et al.*, 2009:1723). After obtaining the R/S the H is estimated by determining the slope of a linear regression with $\log(R/S_n)$ as the dependent variable and the log of the window length *n*. *H* is, therefore, estimated by running an OLS regression:

$$\log(R/S_n) = \alpha + H\log(n) \tag{5}$$

If $0<H<1$ the time series exhibits long memory and *d* is calculated by subtracting 0.5 from *H* (Hosking, 1981:167). The above explained method forms the most basic of the rescaled analysis methods. The R/S analysis has been shown to be superior to methods such as:

i) the analysis of autocorrelations,

ii) variance ratios, and

iii) spectral decompositions (Mandlebrot & Wallis, 1969a; Mandelbrot, 1972; Mandelbrot & Taqqu, 1979).

The findings reported above may be contested, but there is no doubt about the ability of R/S analysis to determine the long memory in time series (Lo, 1991:1288). A significant short-coming of the R/S analysis is its large measure of sensitivity to short-range dependence. The latter implies that the R/S statistic might be a product of the short-term memory of a series rather than that of the long-term memory properties of the particular series.

Lo (1991:1289) suggested a new R/S analysis which provides the econometrician with the manner of distinguishing between short- and long-term dependence. As a result, the R/S statistic is modified to change due to long memory processes and be independent of short memory processes. This method of rescaled range analysis has since been referred to as the modified R/S statistic (MRS) (Lo, 1991:1289 and Erfani *et al.*, 2009:173). MRS is denoted by R'/S_n and is defined as:

$$R'/S_n \equiv \frac{1}{\sigma_n(q)}\left[\underset{0\le k\le n}{Max}\sum_{j=1}^{k}(X_t-\bar{X}_n)-\underset{0\le k\le n}{Min}\sum_{j=1}^{k}(X_t-\bar{X}_n)\right] \tag{6}$$

where:

$$\sigma_n^2(q)\equiv\frac{1}{n}\sum_{j=1}^{n}(X_j-\bar{X})^2+\frac{2}{n}\omega_j(q)\left[\sum_{i=j+1}^{n}(X_i-\bar{X}_n)(X_{i-j}-\bar{X}_n)\right], \tag{7}$$

$$\begin{aligned}&\sigma_n^2(q)\equiv\sigma_x^2+2\sum_{j=1}^{q}\omega_j(q)\gamma_j,\\&\omega_j(q)\equiv1-\frac{j}{q+1},\\&q<n.\end{aligned} \tag{8}$$

The sample auto covariance and variance of X in the above equations are γ_j and σ_n^2 respectively. The difference between R/S in Equation 3 and the MRS in Equation 6 lies in the denominator. On the one hand, the R/S statistic is calculated by using the corresponding window's standard deviation, while on the other hand, the MRS is calculated by using the square root of the window's estimated variance and the weighted autocovariance up to lag *q* (Lo, 1991:1290; Erfani *et al.*, 2009:173). Following the calculation of the MRS for different windows of length *n*, the analysis follows that of the R/S statistic. Therefore, using the method of OLS, the Equation 9 is regressed:

$$\log(R'/S_n)=\alpha+H\log(n) \tag{9}$$

The slope of the regression represents the *H*-coefficient which indicates the presence of long memory if $0<H<1$. Furthermore, *d* is calculated by subtracting 0.5 from *H* (Hosking, 1981:167). Hosking (1981:175) devised, as an alternative to the methods discussed above, a maximum likelihood method may be used to estimate *d* (Hosking, 1981:175).

After calculating *d* it is used to reduce a non-stationary financial time series to a stationary financial time series by fractionally differencing the series with *d*. This method of using MRS in calculating *H*, *d* and fractionally differencing to achieve stationarity was used by Erfani *et* *al*. (2009:1724) on a time series of the daily closing prices of the Tehran Stock Exchange index.

Since a financial time series may be integrated to the order of *I(d)*, with *d* being a non-integer value the time series is fractionally integrated and may be reduced to stationarity by fractionally differencing the series by *d* (Burke *et al*., 2005:31). After establishing *d*, the fractionally differenced financial time series, w_t, is obtained as follows:

$$w_t = (1-L)^d x_t, \tag{10}$$

where, w_t is the fractionally differenced financial time series, x_t is the financial time series in levels, *d* is the fractional differencing parameter, L is the lag operator, and $(1-L)^d$ is the fractional difference operator defined as:

$$(1-L)^d \equiv \sum_{k=0}^{\infty} \frac{\Gamma(k-d)}{\Gamma(k+1)\Gamma(-d)} L^k. \tag{11}$$

Applying Equation 10 to non-stationary financial time series may reduce the particular time series to a fractionally differenced stationary financial time series (Erfani *et al*., 2009:1724).

In addition to differencing a non-stationary time series may consist of equal variations, such as cyclical or seasonal variations, surrounding a deterministic trend. The presence of equal variations around the deterministic trend may indicate the presence of a stationary component being present in the time series. Therefore, by de-trending or filtering the time series it will produce a time series that is stationary and has become stationary by not applying any method of differencing. The following section of this literature review reports the topics of trends, filtering and achieving stationarity without differencing.

3.4 Trend, seasonal and cyclical stationarity

Enders (2010:189) explains the difference between a time series containing a trend and a stationary time series by referring to the influences of shocks on a series. A stationary time series will only be affected temporarily by shocks and as time passes the series will revert back to its long-run mean. However, a shock to a series containing a trend will cause a series to deviate from its mean and not return to its long-run level.

A financial time series may be non-stationary due to either a deterministic trend or a stochastic trend (Burke *et al*., 2005:30). A time series containing a deterministic trend may be de-trended to remove the trend and a series that contains a stochastic trend may be differenced to remove the trend (Enders, 2010:257). As a result, differencing is not an appropriate method for removing a deterministic trend and de-trending is not an appropriate manner of attempting to remove a stochastic trend (Enders, 2010:257). The importance of determining the type of trend present in financial time series, before simply differencing the series to obtain stationarity, is therefore, highlighted by the latter. A non-stationary time series containing a stochastic trend and that is integrated of order one, *I(1)*, is reduced to stationarity by differencing the series and is known as a difference stationary time series (Clements and Hendry, 2001:S1). On the other hand, a trend stationary time series refers to a non-stationary time series that is rendered stationary only by de-trending the time series (Clements *et al.*, 2001:S1).

A review of the literature and empirical study surrounding time series analysis reveals an array of different de-trending methods available to the time series analyst (e.g. Kalman, 1960, Hodrick and Prescott, 1997 and Baxter and King, 1999). The methods established by these authors ranged from the simple methods presented by Enders (2010:191) to the more complex methods presented by Hodrick *et al.* (1997). The most popular methods used by econometricians to de-trend macroeconomic time series are the Hodrick-Prescott (HP) filter and the Baxter-King approximate bandpass (BK) filter (Aadland, 2002:2).

The process of de-trending financial time series containing a deterministic trend is explained by Enders (2010:191) as follows: consider a time series consisting of a deterministic trend and a pure noise component:

$$y_t = y_0 + a_1 t + \varepsilon_t, \tag{12}$$

where y_0 refers to the initial condition for period zero, $a_1 t$, is the deterministic trend component and ε_t is the pure noise component. The time series, y_t, is de-trended by regressing Equation 12 and obtaining the values of the series ε_t by subtracting the estimated values of y_t from the observed values in the time series. Alternatively, a financial time series may consist of a deterministic polynomial trend:

$$y_t = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \ldots + a_n t^n + e_t, \tag{13}$$

where e_t represents a stationary process. Under these circumstances de-trending is achieved by means of regression with y_t as dependent variable and a deterministic polynomial time trend as independent variable. The *t*-tests, *F*-tests, and/or the Akaike Information Criterion (AIC) or Schwartz Bayesian Criterion (SBC) statistics are used to determine the correct degree of the polynomial.[8] The stationary and de-trended series e_t is the product of subtracting the estimated values of y_t from the observed values of y_t. The stationary de-trended time series yielded from the above methods may now be used in models used for time series analysis (Enders, 2010:191). Another method of de-trending employs certain unit root tests with stationarity as alternative or by using filters. Unit root tests form the central point of discussion in the following section of this paper while filters are discussed next.

Aadland (2005:290) explains that the HP filter is widely used in the de-trending of macroeconomic time series. In the HP filter there is a trade-off between the squared deviations from a trend and a smoothness constraint. The HP filter is given by Hodrick *et al.* (1997)as:

$$h(L) = \frac{\lambda(1-L)^2(1-L^{-1})^2}{1+\lambda(1-L)^2(1-L^{-1})^2}, \tag{14}$$

where λ is an adjustable smoothness parameter. Apart from the HP filter, another widely used filter is the BK filter which is based on the theory of spectral band-pass filters. These filters may be used to remove a trend from a cyclical stationary component in a time series which is non-stationary due to the trend component.

Time series may also consist of a seasonal component, rather than a cyclical component, fluctuating around a trend. Montgomery *et al*. (2008:39) suggested the following for seasonal differencing:

$$\nabla_d y_t = (1-B^d) = y_t - y_{t-d}, \tag{15}$$

where the lag-*d* is the seasonal difference operator. If a trend remains after seasonally differencing the data, Montgomery (2008:39) suggest continuing by first differencing the data.

[8] For a comprehensive explanation on how the *t*-tests, *F*-tests, and/or the Akaike Information Criterion (AIC) or Schwartz Bayesian Criterion (SBC) statistics are used to determine the correct degree of the polynomial, refer to Enders (2010:191).

Up to this point, the different methods of rendering a financial time series stationary, have been discussed. Subsequently it is necessary to confirm stationarity by conducting a test for stationarity. The different tests used to confirm that a financial time series is present is discussed in the following section.

4. The testing of stationarity in financial time series

Testing stationarity in financial time series involves testing for the order of integration in the time series and therefore, whether the time series possess a unit root. There are two principal tests popular amongst econometricians for testing the null-hypothesis of a unit root to establish stationarity. These are the Augmented Dickey-Fuller (ADF) test for unit roots (a modification of the original Dickey-Fuller (DF) test) and the Phillips-Perron test (Asteriou *et al*., 2007:297). There are also several tests for testing stationarity with stationarity as the null-hypothesis. A popular such test is the KPSS test (after Kwaitkowski, Phillips, Schimdt and Shin, 1992). Other tests with stationarity as null include: Tanaka (1990), Park (1990), Saikkonen and Luukkonen (1993), Choi (1994), the Leybourne and McCabe test (1994) and Arellano and Pentula (1995).

4.1 Unit root tests

Dickey and Fuller (1979, 1981) developed a procedure for testing non-stationarity based on the presence of a unit root. This procedure has become known as the Dickey-Fuller test (DF) for unit roots (Asteriou *et al*., 2007:295). The DF test provides the econometrician with three different regressions that may be used to test for the presence of a unit root (Enders, 2010:206):

$$\Delta y_t = \gamma y_{t-1} + \varepsilon_t \tag{16}$$

$$\Delta y_t = a_0 + \gamma y_{t-1} + \varepsilon_t \tag{17}$$

$$\Delta y_t = a_0 + \gamma y_{t-1} + a_2 t + \varepsilon_t \tag{18}$$

The difference between Equations 16 to 18 is the presence of intercepts and a linear time trend, whereas Equation 17 includes an intercept and Equation 18 includes an intercept and a deterministic time trend. According to Asteriou *et al*. (2007:296) and Enders (2010:206) γ is the central focus point of all three equations and if γ = 0, the time series contains a unit root.[9] An alternative to the DF test is the Augmented Dickey-Fuller (ADF) test for stationarity.

In the unlikely event of a white noise error term, Dickey and Fuller extended the DF test to include extra lagged terms of the explanatory variable (Asteriou *et al.*, 2007:297). This extended version is known as the ADF test and the inclusion of extra lagged dependent variables eliminates the presence of autocorrelation. The ADF follows on the three different forms used in the DF test and can be conducted using the following three regressions:[10]

$$\Delta y_t = \gamma y_{t-1} + \sum_{i-1}^{p} \beta_i \Delta y_{t-i} + \varepsilon_t, \tag{19}$$

$$\Delta y_t = a_0 + \gamma y_{t-1} + \sum_{i-1}^{p} \beta_i \Delta y_{t-i} + \varepsilon_t, \tag{20}$$

$$\Delta y_t = a_0 + \gamma y_{t-1} + a_2 t + \sum_{i-1}^{p} \beta_i \Delta y_{t-i} + \varepsilon_t. \tag{21}$$

Asteriou (2007:295) explains a three step method for using the DF and ADF tests for unit roots with the aim of concluding that a series is stationary. The first is to test the time series for a unit root and if none exists, the time series is stationary and *I(0)*, otherwise it contains a unit root and is *I(n)*. The second step is to take first differences and testing the first differenced series for a unit root and, if there is none, it is stationary and *I(1)*, otherwise it contains a unit root and is *I(n)*. The third step entails differencing the time series up to the point where the tests indicate no presence of a unit root. In the later step the time series will be integrated to the order of times needed to difference the time series.

Phillips and Perron (1988) developed the Phillips-Perron (PP) test similar to the ADF test. The difference between the PP and ADF is that the while ADF corrects for autocorrelation by adding lagged values of the dependent variable. The PP test accounts for autocorrelation in e_t by making a correction to the *t*-stat of γ from the AR(1) regression. The PP test regression is:

$$\Delta y_{t-1} = a_0 + \gamma y_{t-1} + e_t. \tag{22}$$

[9] See Enders (2010:206), Asteriou *et al*. (2007:296) and Maddala *et al*. (2000:61) for an in-depth explanation of the DF test.
[10] See Enders (2010:215), Asteriou *et al*. (2007:297) and Maddala *et al*. (2000:75) for an in-depth explanation of the DF test.

The DF, ADF and PP tests have been criticised for its lack of power in determining the presence of unit roots. More particularly, most of the criticism originates from these tests being useless when working with data frequencies greater than quarterly (Maddala *et al.*, 2000:45&92). This poses a problem for the econometrician analysing financial time series data which tends to be either daily or intra-day data. The ability of these tests to determine fractional unit roots are also unknown and poses a further problem for researchers aiming to determine whether a time series is fractionally differenced stationary after fractionally differencing the time series.

4.2 Test with stationarity as null-hypothesis

While tests for unit roots with stationarity as an alternative are popular amongst econometricians, there are also tests available that have stationarity as the null hypothesis and a unit root as the alternative. The most popular amongst these tests is the KPSS test (Maddala *et al.*, (2000:120). The KPSS test was developed by Kwaitkowski, Phillips, Schimdt and Shin (1992) and incorporates the following model:

$$y_t = \delta_t + \zeta_t + \varepsilon_t, \tag{23}$$

where ε_t represents a stationary process and ζ_t is a random walk specified as:

$$\zeta_t = \zeta_{t-1} + u_t, \quad u_t \sim iid(0, \sigma_u^2), \tag{24}$$

and u_t is an independently and identically distributed (iid) error term.

The null hypothesis is that of stationarity:

$H_0 : \sigma_u^2 = 0$ or ζ_t is a constant.

The Nabeya-Tanaka test statistic, also known as the LM test, for the hypothesis of the KPSS test is specified as:

$$LM = \frac{\sum_{t=1}^{T} S_t^2}{\sigma_e^2}, \tag{25}$$

where y_t is regressed on a constant and a time trend, where ε_t are the residuals. The residual variance of the regression is given by σ_e^2 and s_t is the partial sum of ε_t defined by:

$$S_t = \sum_{i=1}^{t} e_t,\ \ t = 1, 2, ..., T. \quad (26)$$

To determine whether the time series is stationary in levels, rather than testing trend stationarity, the test is conducted with a regression of y_t on an intercept. Nabeya and Tanaka derived the asymptotic distribution of the LM test statistic and the critical values are presented in Kwaitkowski *et al*. (1992:166)(Maddala *et al*., 2000:121).

4.3 The Dickey-Fuller test with GLS de-trending (DF-GLS)

As a method of overcoming the low power of the DF, ADF and PP unit root tests, Maddala *et al*. (2000:114) suggests using the DF-GLS test. The DF-GLS is a modification to the ADF and was proposed by Elliott, Rothenberg, and Stock (1996) and is denoted by ERS. The DF-GLS de-trends a time series y_t and produces a series y_t^d which replaces y_t in the original ADF test equation and is given as:

$$\Delta y_t^d = \gamma y_{t-1} + \sum_{i-1}^{p} \beta_i \Delta y_{t-i}^d + \varepsilon_t. \quad (27)$$

The critical values of the test statistic are provided in ERS (1996:825).

The discussion above includes a wide variety of tests available to the econometrician performing analysis on financial time series. The choice of which test to use is therefore made difficult. Kwaitkowski *et al.* (1992:176) and Choi (1994:721) suggests that it would be wise to use a combination of the tests discussed above. One such combination can comprise the use the ADF and KPSS tests and is referred to as confirmatory analysis (Maddala *et al*., 2000:126). Stationarity can be deduced if the null-hypothesis of the one test is rejected and the null-hypothesis of the other is not. The reason for this is that the null for the ADF is that of a unit root while the null of the KPSS is that of stationarity.

5. Conclusion

The stationarity of time series is an essential characteristic required to be achieved before analysing a time series. Stationarity has been shown to be a prerequisite for performing applied econometric analysis. It has become common practice to difference financial time series data in order to achieve stationary. This paper challenges the common practice of differencing data indiscriminately by presenting an overview of what has been established through other methods of achieving stationarity. These methods include achieving stationarity through fractional differencing and de-trending.

A variety of different forms of stationarity that can occur while using financial time series has been presented. These include: 1) difference stationary, 2) fractionally differenced stationary, 3) trend stationary, and 4) cyclical and seasonal stationary. An important factor in rendering a time series stationary is to determine the presence of a stochastic trend, deterministic trend, unit root, cyclical component or seasonal component. In the case of determining the presence of a stochastic trend or unit root, the econometrician must follow the differencing method for achieving stationarity. However, should a time series display a deterministic trend, the data must be de-trended and then tested for stationarity before any differencing can take place. While a time series containing a cyclical component can be de-trended by using the HP filter or BK filter to determine whether the series is cyclical stationary. A seasonal stationary time series can be achieved by removing a seasonal component from that time series by seasonal differencing. After a financial time series has been reduced to stationarity it is necessary to confirm stationarity through tests. The popular tests available, that has a unit root as null hypothesis with stationarity as an alternative, are the DF, ADF, PP and DF-GLS. The econometrician can also use the KPSS test; the null hypothesis here is stationarity. It has been suggested that the econometrician should use the KPSS test in conjunction with one of the unit root tests as a confirmatory analysis for the presence of stationarity.

The discussion above shows the econometrician and financial analyst that, reducing a financial time series to stationarity, is a much more integrate process than simply taking first difference, testing for stationarity with the ADF test, and proceeding with, for example, forecasting models. Therefore, when conducting applied econometric analysis to time series it is necessary to investigate the properties of the time series before launching into differencing the data. Only thereafter should it be decided which method must be applied to reduce a financial time series to the correct form of stationarity.

References

AADLAND, D. 2005. De-trending time-aggregated data. *Economic Letters*, 89:287-93.

ARELLANO, C. & PENTULA, S.G. 1995. Testing for trend stationarity versus difference stationarity. *Journal of Time Series Analysis*, 16:147-164.

ASTERIOU, D. & HALL, S.G. 2007. Applied econometrics: a modern approach. New York: Palgrave Macmillan. 397 p.

BAXTER, M. & KING, R.G., 1999. Measuring business cycles: approximate band-pass filters for economic time series. *Review of Economics and Statistics*, 81:575–593.

BOX, G.E.P. & JENKINS, G.E.M. 1976. Time series analysis: Forecasting and control. San Francisco: Holden Day. 575 p.

BURKE, S.P. & HUNTER, J. 2005. Modelling non-stationary economic time series: A multivariate approach. New York: Palgrave MacMillan. 253 p.

CACCIA, D.C., PERCIVAL, D., CANNON, M.J., RAYMOND, G. & BASSINGWAIGHTE, J.B. 1997. Analyzing exact fractal time series: Evaluating dispersional analysis and rescaled range methods. *Physica A*, 246(3-4):609-632.

CHOI, I. 1994. Residual-based tests for the null of stationarity with application to US macroeconomic time series. *Econometric theory*, 10:720-746.

DAVIES, R.B. & HARTE, D.S. 1987. Tests for the Hurst effect. *Biometricka Trust*, 74(1):95-101. Mar.

DICKEY, D.A. & FULLER, W.A. 1979. Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American Statistical Association*, 74(366):427-431. Jun.

DICKEY, D.A. & FULLER, W.A. 1981. Likelyhood ratio statistics for autoregressive time series with a unit root. *Econometrica*, 49(4):1057-172. Jul.

ENDERS, W. 2010. Applied econometric time series. Hoboken: Wiley. 517 p.

ERFANI, A. & SAMIMI, A.J. 2009. Long memory forecasting of stock price index using a fractionally differenced ARMA model. *Journal of Applied Sciences Research*, 5(10):1721-1731.

FAMA. E.F. 1970. Efficient capital markets: A review of theory and empirical work. *The Journal of Finance*, 25(2):383-417. May.

FIELITZ, B.D. 1971. Stationarity of random data: Some implications for the distribution of stock price changes. *Journal of Financial and Quantitative Analysis*, 6(3):1025-1034.

GUJARATI, D.N. 2006 Essentials of econometrics. New York: Mc Graw Hill. 553 p.

HODRICK, J.R. & PRESCOTT, E.C. 1997. Postwar U.S. business cycles: An empirical investigation. *Journal of Money, Credit and Banking*, 29(1):1-16. Feb.

HOSKING, J.R.M. 1981. Fractional differencing. *Biometrika trust*, 68(1):1025-1034.

HURST, H.E. 1951. Long-term storage capacity of reservoirs. *Transactions of the American Society of Civil Engineers*, 116:770-799.

KALMAN, R.E. 1960. A new approach to linear filtering and prediction problems. *Journal of Basic Engineering*, 82(1): 35–45.

KING, R.G. & REBELO, S.T. 1993. Low frequency filtering and real business cycles. *Journal of Economic Dynamics and Control*, 17:207–231.

KWAITKOWSKI, D. PHILLIPS, P.C.B., SCHMIDT, P. & SHIN, Y. 1992. Testing the null hypothesis of stationarity against the alternative of a unit root. *Journal of Econometrics*, 54:159-178.

LEYBOURNE, S.J. & MCCABE, B.P.M. 1994. A consistent test for a unit root. *Journal of Business and Economic Statistics*, 12:157-166.

LEYBOURNE, S.J., MCCABE, B.P.M. & TREMAYNE, A.R. 1996. Can economic time series be differenced to stationarity? *Journal of Business & Economic Statistics*, 14(4):435-446. Oct.

LO, A.W. 1991. Long-term memory in stock market prices. *Econometrica*, 59(5):1279-1313. Sep.

MADDALA, G.S. & KIM, I.M. 2000. Unit roots, cointergration, and structural change. Cambridge: Cambridge. 505 p.

MANDELBROT, B. & TAQQU, M. 1979. Robust R/S analysis of long-run serial correlation. *Bullitin of the International Statistical Institute*, 42nd Conference Proceedings, Manila. 2:59-104.

MANDELBROT, B. & WALLIS, J. 1969a. Computer experiments with fractional Gaussian noises. *Water Resources Research*, 5:228-267. Feb.

MANDELBROT, B. 1972. Statistical methodology for non-periodic cycles: From the covariance to R/S analysis. *Annals of Statistics*, 23(3):1015-1028.

MCCABE, B.P.M. & TREMAYNE, A.R. 1995. Testing time series for difference stationarity. *The Annals of Statistics*, 23(3):1015-1028.

MONTGOMERY, D.C., JENNINGS, C.L. & KULAHCI, M. 2008. Introduction to time series analysis and forecasting. Hoboken: Wiley. 445 p.

PARK, J.Y. 1990. Testing for unit roots and cointegration by variable addition. *Advances in Econometrics*, 8:107-133.

PENG, C.K., HAVLIN, S., STANLEY, H.E. & GOLDBERGER, A.L. 1994. Quantification of scaling exponents and crossover phenomena in non-stationary heartbeat time series. *Chaos*, 5(1):82-87.

SAIKKONEN, P. & LUUKKONEN, R. 1993. Testing for a moving average unit root in ARIMA models. *Journal of the American Statistical Association*, 88:596-601.