(PRELIMINARY DRAFT)

The Terms of Trade and the Resource Curse

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Abstract

This paper explores how the level and volatility of the terms of trade of resource-abundant countries might partly explain their poor economic performance. The terms of trade are incorporated in a stochastic growth model, from which the effects of the trend level and volatility of the terms of trade on saving and economic growth are derived. It is shown that if the coefficient of relative risk aversion is sufficiently small, a decreased average growth rate and an increased volatility of the terms of trade both decrease the propensity to save and economic growth. If the coefficient of relative risk aversion is sufficiently large, on the other hand, the model predicts that a decreased average growth rate and an increased volatility of the terms of trade both increase the propensity to save and economic growth.

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1 Introduction

In a seminal paper, Sachs and Warner (1995) show that resource-rich countries have generally experienced slower economic growth than resource-poor countries. A subsequent wealth of research has offered several explanations for this phenomenon, with empirical analyses suggesting that the 'resource curse' is the result of some combination of them. The explanations tend to fall into three broad categories: those pertaining to prevention of industrialisation, those pertaining to political economy, and those pertaining to commodity prices.

This paper concerns itself with the final class of explanations, those which suggest that certain characteristics of natural resource prices have a negative effect on economic growth. Both the level and volatility of these prices, and by implication the terms of trade (TOT) of resource-abundant countries, have been cited as potential channels of the resource curse.

An increase in a country's TOT is generally thought, *ceteris paribus*, to be favourable for growth, since it serves to increase that country's purchasing power over imports. By the same reasoning, a decrease in the TOT should be bad for growth. Secular trends in the TOT of resource-intensive countries would thus be likely to influence the growth paths of those countries.

The Prebisch-Singer thesis contends that the prices of primary commodities follow a downward trend in the long run, relative to the prices of manufactures and other products (Prebisch, 1950; Singer, 1950). Countries that specialise in primary products will thus tend to experience declining TOT in the long run. The rationale is that world demand for primary products is inelastic with respect to world income, so that an increase in world income is accompanied by a proportionately smaller increase in demand for primary products. If true, this would suggest that resource-based growth would eventually be choked by resources becoming relatively less and less valuable in the world market.

Others, however, have argued that the price of natural resources should theoretically follow an *upward* trend, since their total supply is fixed while the world's population is continuously increasing (Malthus, 1798; Hotelling, 1931). Empirical studies have disagreed on whether the overall long-run trend of the TOT of commodity producers has been upward or downward, but they have been consistent in suggesting that there have been both long periods of downward trends as well as long periods of upward trends (Frankel, 2010). Whatever the case empirically, the TOT-level channel through which the resource curse is generally thought to operate is the negative effect declining TOT have on growth, an hypothesis we shall test in this paper.

Commodity prices have also tended to be far more volatile in the short and medium runs than the prices of manufactures and other products, in turn causing the TOT of resource producers to be volatile (Deaton and Laroque, 1992; Deaton and Miller, 1996; Combes and Guillaumont, 2002; Pindyck, 2004). This can be harmful to growth for a number of reasons. TOT volatility can depress savings and investment by increasing both investment risk and the option value of delaying investment (Dixit and Pindyck, 1994). Also, temporary TOT shocks have often been misread by governments as permanent, resulting in irreversible spending commitments that have led to macroeconomic imbalances (Little et al., 1993).

Whatever the channels, empirical analyses have consistently noted the importance of both the level and volatility of TOT in explaining output growth rates. Easterly et al. (1993), Fischer (1993), Barro and Sala-i-Martin (1995) and Blattman et al. (2007) all find that decreases in the level of the TOT are associated with lower growth rates of output, while Lutz (1994) and Ramey and Ramey (1995) find a negative association between TOT volatility and output growth. It is odd, then, that there is a paucity of literature attempting to formally characterise the link between the TOT and economic growth.

A notable exception is found in Mendoza (1997), where the TOT are endogenised in a stochastic growth model. As shown in Section 2 of this paper, Mendoza's model predicts that the mean rate of change of TOT is positively linked with consumption growth, while the effect of TOT volatility on consumption growth depends on the degree of risk aversion. Mendoza's analysis has two limitations, however. The first is that it does not explicitly consider the effects of the TOT, but rather an 'effective rate of return' comprising the change in TOT and an interest rate. The second, noted by Bleaney and Greenaway (2001), is that its predictions concern growth in consumption, rather than output growth, so that they do not in actual fact characterise the link between TOT and output growth. As a result, the Mendoza model cannot explain the TOT channels of the resource curse.

This paper extends Mendoza's analysis by addressing these two limitations. The effective rate of return is separated into the TOT and an interest rate, so that the effects of the former may be analysed more explicitly. This technical augmentation also allows us to consider the effects of the TOT on *output*, so that a TOT channel through which the resource curse might operated is formally modelled. It will be shown that the crucial parameter in this regard is the country's coefficient of relative risk aversion; for certain values of this parameter, the resource curse operates as expected (decreased TOT level and increased TOT volatility hamper output growth), while for other values the opposite effect occurs, and the TOT-channels of the resource curse that are traditionally thought to negatively influence growth actually do the opposite.

The structure of the paper is as follows: Section 2 presents Mendoza's model and derives its implications, before presenting an augmentation of the model to explicitly take into account the level and volatility of the TOT, further allowing the model's implications to extend to output growth. The implications of Mendoza's model and our augmentation thereof are listed as 'Predictions'. Section 3 is a discussion of the implications of the model for the resource curse. Section 4 concludes.

2 The model

We follow Mendoza (1997) in using a basic stochastic neoclassical model that extends the savings-under-uncertainty framework developed by Phelps (1962) and Levhari and Srinivasan (1969) to include TOT shocks. A small open economy is inhabited by households who formulate optimal plans for the consumption of an imported good so as to maximise the present value of expected lifetime utility:

$$U(c_0, c_1, \ldots) = E\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right], \quad 0 < \beta < 1, \tag{1}$$

where c_t is period-t consumption of the imported good and β is the discount factor. The utility function is of the form

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}, \quad \gamma > 0, \tag{2}$$

where γ is the coefficient of relative risk aversion.

The production technology is linear, and takes the form of a perfectly durable asset that yields a stochastic return each period. The return is an exportable commodity, which agents can exchange for the imported consumable in a perfectly competitive world market. The intertemporal resource constraint is given by

$$A_{t+1} = R_t(A_t - p_t c_t), \tag{3}$$

where A_t is the stock of wealth in terms of exportables, p_t is the relative price of imports in terms of exports (so that the terms of trade $TOT_t = 1/p_t$), and R_t is the domestic gross rate of return on savings. The effective rate of return is given by $r_t = R_t p_t / p_{t+1}$; we assume that the r_t are independent and identically distributed, with that distribution known.

The competitive equilibrium is defined by the optimal intertemporal consumption plan, which maximises (1) subject to (3). The optimality conditions are the constraint and the intertemporal Euler equation:

$$U'(c_t) = \beta E\left[r_t U'(c_{t+1})\right]. \tag{4}$$

Lemma 1. Closed-form solutions for the model are given by:

$$c_t^* = \lambda \left(\frac{A_t}{p_t}\right), \tag{5}$$

$$A_{t+1}^* = (1-\lambda) R_t A_t, \tag{6}$$

where

$$\lambda \equiv 1 - \beta^{1/\gamma}\left[E(r_t^{1-\gamma})\right]^{1/\gamma}, \tag{7}$$

subject to the feasibility condition

$$E(r_t^{1-\gamma}) < \beta^{-1}. \tag{8}$$

A proof of this result is contained in the appendix.

We assume the feasibility condition to hold, so that consumption in each period is a positive fraction λ of the real value of asset wealth in terms of importables. Note that p_t enters the consumption plan but R_t does not, because the realisation of the former is known when c_t is chosen while that of the latter is not.

We have assumed that the r_t are independent and identically distributed, with that distribution known. We now make the further assumption that r_t follows a log-normal i.i.d. distribution:[1] $\ln r_t \sim N(\mu, \sigma^2)$. We then have

[1] The basis of this assumption is empirical. Using data from 40 industrial and developing countries over the period 1971-1991, Mendoza (1997) shows that the data cannot reject the hypothesis that both the effective rate of return and the proportional rate of change of TOT follow log-normal distributions.

(i) $\mu_r \triangleq E(r_t) = \exp\left[\mu + \sigma^2/2\right]$

(ii) $\sigma_r^2 \triangleq var(r_t) = \mu_r^2\left(e^{\sigma^2} - 1\right)$

(iii) $E(r_t^{1-\gamma}) = \exp\left[\mu(1-\gamma) + (1-\gamma)^2\sigma^2/2\right]$.

Substituting the above results into (7) allows us to write the marginal propensity to consume (and therefore the savings rate s as well) in terms of μ_r and σ^2:

$$\lambda(\mu_r, \sigma^2) \equiv 1 - \left(\beta\mu_r^{1-\gamma}\right)^{1/\gamma} \exp\left(-(1-\gamma)\frac{\sigma^2}{2}\right) \tag{9}$$

$$s(\mu_r, \sigma^2) \equiv 1 - \lambda \equiv \left(\beta\mu_r^{1-\gamma}\right)^{1/\gamma} \exp\left(-(1-\gamma)\frac{\sigma^2}{2}\right). \tag{10}$$

The partial derivatives of $\ln s$ with respect to μ_r and σ^2 are then[2]

$$\frac{\partial \ln s}{\partial \mu_r} = \frac{1-\gamma}{\gamma\mu_r} \tag{11}$$

$$\frac{\partial \ln s}{\partial \sigma^2} = -(1-\gamma)/2, \tag{12}$$

which yield some preliminary predictions of the model.

Prediction 1: An increase in μ_r will increase the savings rate if $\gamma < 1$, but will decrease the savings rate if $\gamma > 1$.

Prediction 2: A mean-preserving increase in σ^2 will decrease the savings rate if $\gamma < 1$, and will increase it if $\gamma > 1$.

Prediction 1 states that, if the coefficient of relative risk aversion is sufficiently low, an increase in the average effective rate of return on savings will increase the propensity to save. On the other hand, if the coefficient of relative risk aversion is higher than unity, an increase in the average effective return on savings will increase the propensity to consume. Alternatively, if two countries share a coefficient of relative risk aversion below unity, the country with the higher average effective rate of return will have the higher savings rate, *ceteris paribus*; the order is reversed if the shared coefficient of relative risk aversion is greater than unity.

The intuition is that an increase in the average rate of return induces two opposing consumption inclinations. The first is an inclination to save more to take advantage of the higher return; the second is an inclination to consume more, expecting higher future wealth, to smooth consumption over time. Whether the net effect is a higher or lower savings rate depends on the parameter γ.

The second prediction states that an increase in the volatility of the effective rate of return will decrease the savings rate if the coefficient of relative risk

[2] Since feasibility requires $s > 0$, the sign of each derivative of $\ln s$ is the same as the corresponding derivative of s:

$$\frac{\partial \ln s}{\partial\, \cdot} = \frac{\partial \ln s}{\partial s}\frac{\partial s}{\partial\, \cdot} = \frac{1}{s}\frac{\partial s}{\partial\, \cdot}$$

so that $\mathrm{sgn}(\frac{\partial \ln s}{\partial\, \cdot}) = \mathrm{sgn}(\frac{1}{s}\frac{\partial s}{\partial\, \cdot}) = \mathrm{sgn}(\frac{\partial s}{\partial\, \cdot})$.

aversion is sufficiently low, but will increase it if the coefficient of relative risk aversion is sufficiently high. The cross-country interpretation is that, given two countries with the same coefficient of relative risk aversion less than one, the country whose effective rate of return is subject to higher volatility will have the lower savings rate *ceteris paribus*, and vice-versa if the shared coefficient of relative risk aversion is greater than unity.

Again, the intuition lies in the existence of two opposing consumption inclinations in the face of higher risk. The first is an inclination to save less in order to expose a lower level of savings to the return risk; the second is an inclination to save more to protect against potentially low future wealth. Once again, γ determines the net effect of the two.

2.1 Consumption growth

We now employ a first-difference filter to separate the trend and cyclical components of consumption. This will allow us to examine the effect of both the level and volatility of r_t on consumption growth. We begin by using (5) and (6) to write

$$\frac{c_{t+1}}{c_t} = (1-\lambda)r_t = s\,r_t. \tag{13}$$

Denote the log first-difference of consumption by $\Delta \ln c_t \triangleq \ln c_t - \ln c_{t-1} = \ln(c_t/c_{t-1})$. We proceed by writing $\ln r_t \equiv \mu + \epsilon_t$, so that ϵ_t is the period-t deviation of $\ln r_t$ from its mean. ϵ_t is thus an i.i.d. normal process with zero mean and a variance of σ^2. It follows from (13) that

$$\begin{aligned}\Delta \ln c_t &= \ln s + \ln r_{t-1} \\ &= \underbrace{\frac{1}{\gamma}\big(\ln\beta + \ln\mu_r\big) - \big(2-\gamma\big)\frac{\sigma^2}{2}}_{\text{Trend component}} \quad + \underbrace{\epsilon_{t-1}}_{\text{Cyclical component}}. \end{aligned} \tag{14}$$

Approximating the growth rate of consumption by $\Delta \ln c_t$, and noting that

$$\frac{\partial\, \Delta \ln c_t}{\partial \mu_r} = \frac{1}{\gamma\mu_r} \tag{15}$$

$$\frac{\partial\, \Delta \ln c_t}{\partial \sigma^2} = -(2-\gamma)/2, \tag{16}$$

we have the following predictions:

PREDICTION 3: An increase in μ_r increases the average growth rate of consumption irrespective of the value of γ.

PREDICTION 4: A mean-preserving increase in σ^2 decreases the average growth rate of consumption if $\gamma < 2$, but increases it if $\gamma > 2$.

Prediction 3 is a strong result: the growth rate of consumption will increase in the face of an increase in the average effective rate of return, regardless of the value of the coefficient of relative risk aversion. Alternatively, given two countries with the same coefficient of relative risk aversion, the country with the larger average effective rate of return will enjoy the larger consumption growth rate *ceteris paribus*.

Recall Prediction 1: if $\gamma < 1$, an increase in the average effective rate of return will induce a decrease in the propensity to consume. Coupled with Prediction 3, this means that the high savings rate generates sufficient growth in wealth (augmented further by the increased average rate of return) to counteract this lower propensity to consume, so that the growth rate in consumption actually increases. On the other hand, for $\gamma > 1$, the increase in the propensity to consume induced by an increase in the average effective rate of return, along with that higher average rate of return, combine to counteract the negative effect of a lower savings rate on growth in wealth, so that the net effect is still an increase in the growth rate of consumption.

Prediction 4 implies that if the degree of risk aversion is sufficiently low, a mean-preserving increase in the volatility of the effective rate of return (i.e. an increase in risk) lowers the average growth rate of consumption. On the other hand, if the degree of risk aversion is sufficiently high, consumption growth rises if risk rises. Alternatively, given two countries with the same $\gamma < 2$, the country whose effective rate of return is subject to higher volatility will suffer a lower average growth rate of consumption, *ceteris paribus*. If the shared γ is greater than two, the opposite result holds. This accords with Prediction 2, as expected, since we have held the average effective rate of return to savings constant in deriving this result.

While the analysis above offers insight into the channels through which the trend and volatility of the TOT might affect aggregate savings and consumption, it has two limitations. First, the model considers the trend and volatility of the effective rate of return, not the TOT directly, so that the effect of the latter is not explicitly modelled. Second, as pointed out by Bleaney and Greenaway (2001), the predictions of the model with respect to consumption growth do not carry over to output (A) growth under the current set of assumptions. Thus, the current model cannot give insight into how the trend and volatility of the TOT might affect output, and as a result it has nothing to say about the TOT channel of the resource curse.

We suggest two responses to the above limitations. The first involves reframing the definition of output to address the second limitation. The second involves making further assumptions about the distributions of the components of r_t, and addresses both limitations. In doing so, it reframes the model as a true growth model, which is then able to characterise the TOT channels through which the resource curse might operate.

2.2 Growth of wealth in terms of importables

Proceeding to the first response, Bleaney and Greenaway's (2001) objection that the predictions of the model do not carry over to output growth is based on writing a similar equation to (13) for wealth in terms of exportables:

$$\frac{A_{t+1}}{A_t} = s\,R_t. \tag{17}$$

Without knowledge of the distribution of R_t, we cannot make predictions about output growth. A simple way to circumvent this problem is to consider output as wealth in terms of importables, $A'_t \triangleq A_t/p_t$. This is arguably a more 'real' definition of wealth, since it takes into account purchasing power. Since $A'_{t+1}/A'_t = C_{t+1}/C_t$, the predictions above with respect to consumption growth *do* carry over to output growth if we consider output as wealth in terms of importables.

While this alteration is appealingly simple, it does not address the first limitation, viz. that the TOT do not explicitly enter the model. To do this, we need to make further assumptions about how the components of the effective rate of return are distributed.

2.3 Growth of wealth in terms of exportables

The second response is more involved, but also further-reaching. We wish to derive a set of predictions regarding the growth rate of wealth in terms of exportables, A. Define $z_t \triangleq p_t/p_{t+1} \equiv TOT_{t+1}/TOT_t$, the proportional growth rate of the terms of trade. Assume that R_t and z_t are independent random variables, both with log-normal i.i.d. distributions,[3] so that

$$\ln R_t \sim N(\bar{\mu}, \bar{\sigma}^2)$$
$$\ln z_t \sim N(\tilde{\mu}, \tilde{\sigma}^2).$$

Since $\ln r_t = \ln R_t + \ln z_t$, $\ln r_t$ is also normally distributed, as before, with

$$\bar{\mu} + \tilde{\mu} \equiv \mu, \;\; \bar{\sigma}^2 + \tilde{\sigma}^2 \equiv \sigma^2, \tag{18}$$

so that

$$\mu_r = \exp\left(\bar{\mu} + \tilde{\mu} + \bar{\sigma}^2/2 + \tilde{\sigma}^2/2\right) = e^{\bar{\mu}+\bar{\sigma}^2/2}\, e^{\tilde{\mu}+\tilde{\sigma}^2/2} = \mu_R\, \mu_z, \tag{19}$$

where $\mu_R \triangleq E(R_t)$ and $\mu_z \triangleq E(z_t)$. Our equation for the savings rate, (10), may be rewritten:

$$s\left(\mu_R, \bar{\sigma}^2; \mu_z, \tilde{\sigma}^2\right) = \left(\beta(\mu_R\, \mu_z)^{1-\gamma}\right)^{1/\gamma} \exp\left[-(1-\gamma)\left(\frac{\bar{\sigma}^2 + \tilde{\sigma}^2}{2}\right)\right], \tag{20}$$

or alternatively:

$$\ln s = \frac{1}{\gamma}\Big[\ln\beta + (1-\gamma)\ln\mu_R + (1-\gamma)\ln\mu_z\Big] - (1-\gamma)\left(\frac{\bar{\sigma}^2 + \tilde{\sigma}^2}{2}\right). \tag{21}$$

We have

$$\frac{\partial \ln s}{\partial \mu_z} = \frac{1-\gamma}{\gamma\mu_z} \tag{22}$$
$$\frac{\partial \ln s}{\partial \tilde{\sigma}^2} = -(1-\gamma)/2, \tag{23}$$

which yield the following predictions:

PREDICTION 5: An increase in the trend level of the proportional growth rate of the TOT will increase the savings rate if $\gamma < 1$, but will decrease the savings rate if $\gamma > 1$.

[3] As noted earlier, this assumption is empirically justified.

Prediction 6: A mean-preserving increase in the volatility of the TOT will decrease the savings rate if $\gamma < 1$, and will increase it if $\gamma > 1$.

These are analogous to Predictions 1 and 2, but are important in their own right in that they incorporate the trend and volatility of the TOT explicitly.

Proceeding as before, we write $\ln R_t = \bar{\mu} + \bar{\epsilon}_t$, so that $\bar{\epsilon}_t$ is the period-t deviation of $\ln R_t$ from its mean, and $\bar{\epsilon}_t$ is an i.i.d. normal process with zero mean and a variance of $\bar{\sigma}^2$. Denote the log first-difference of wealth by $\Delta \ln A_t \triangleq \ln A_t - \ln A_{t-1} = \ln (A_t / A_{t-1})$. It follows from (17) that

$$\begin{aligned}\Delta \ln A_t &= \ln s + \ln R_{t-1} \\ &= \frac{1}{\gamma}\left[\ln \beta + \ln \mu_R + (1-\gamma)\ln \mu_z\right] - \left(2-\gamma\right)\frac{\bar{\sigma}^2}{2} - \left(1-\gamma\right)\frac{\tilde{\sigma}^2}{2} + \bar{\epsilon}_{t-1}.\end{aligned} \tag{24}$$

We approximate the growth rate of A_t by $\Delta \ln A_t$, and note that

$$\frac{\partial\, \Delta \ln A_t}{\partial \mu_z} = \frac{1-\gamma}{\gamma \mu_z} \tag{25}$$

$$\frac{\partial\, \Delta \ln A_t}{\partial \tilde{\sigma}^2} = -(1-\gamma)/2. \tag{26}$$

Prediction 7: An increase in the trend level of the proportional growth rate of the TOT will increase the growth rate of wealth if $\gamma < 1$, but will decrease the growth rate of wealth if $\gamma > 1$.

Prediction 8: A mean-preserving increase in the volatility of the TOT will decrease the growth rate of wealth if $\gamma < 1$, but will increase it if $\gamma > 1$.

3 The resource curse

Predictions 7 and 8 are the main theoretical results of this paper. They give insight into how both the trend level and volatility of the TOT might influence output growth, and thus how the TOT might be a channel of the resource curse.

3.1 The level of the TOT

Prediction 7 states that, if the coefficient of relative risk aversion is sufficiently low, an increase in the average growth rate of the TOT will increase the growth rate of wealth. Similarly, a decrease in the average growth rate of the TOT will negatively affect the growth rate of wealth. On the other hand, if the coefficient of relative risk aversion is sufficiently high, these effects are reversed: an increase in the average growth rate of the TOT has a negative effect on the growth rate of wealth.

Alternatively, given two countries with the same coefficient of relative risk aversion less than unity, the country with the larger growth rate of TOT will experience the higher growth rate of wealth, *ceteris paribus*. Conversely, if the shared coefficient of relative risk aversion is greater than unity, the country with the lower growth rate of TOT will experience the higher growth rate of wealth, *ceteris paribus.*

This ambiguity, resolved only with knowledge of the value of γ, is the result of the presence of two opposing effects induced by an increase in the average rate of return. An increase in the rate of return encourages an agent to increase saving in order to take advantage of the higher returns on savings. But the increased rate of return also increases future wealth; the agent's affinity to smooth consumption over time will encourage her to increase current consumption, thereby reducing current saving. The first effect dominates the second if $\gamma < 1$, while the second effect dominates the first if $\gamma > 1$; the effects balance if $\gamma = 1$.

The implications for a resource abundant country therefore depend both on its coefficient of relative risk aversion and its trend level of TOT. The traditional TOT-level channel of the resource curse postulates that declining TOT of resource-abundant countries negatively affect their economic growth. Our model concurs with this explanation if $\gamma < 1$ and the average proportional growth rate of TOT is low, so that (some form of) the Prebisch-Singer thesis holds. In that case, the lower proportional growth rate of TOT in resource-abundant countries, relative to countries not specialising in resources, decreases their growth rate of wealth relative to the non-resource-abundant countries, all else equal.

On the other hand, a value of γ larger than unity contradicts the traditional explanation of the TOT-level channel of the resource curse. In this case, a lower proportional growth rate of TOT *increases* the rate of economic growth, in contrast to both intuition and empirical evidence.

3.2 The volatility of the TOT

Prediction 8 states that, if the coefficient of relative risk aversion is sufficiently high, an increase in the volatility of the TOT will negatively affect the growth rate of wealth. If the coefficient of relative risk avesion is sufficiently high, on the other hand, such an increase will positively affect growth. The cross-country interpretation is that, given two countries with a shared coefficient of relative risk aversion less than unity, the country whose TOT are subject to higher volatility will experience the lower growth rate of wealth *ceteris paribus*, with the opposite result holding if the shared coefficient of relative risk aversion is greater than unity. Assuming that the TOT of resource-abundant countries are more volatile than those of non-resource countries (as empirical analyses consistently show them to be), the effect of TOT volatility on a resource-abundant country, and thus whether the resource curse operates through this channel, depends again on that country's coefficient of relative risk aversion.

To explain this ambiguity in the effect of volatility on growth, we draw on the 'savings under uncertainty' literature, noting that the uncertainty in our model is of the 'capital risk' type, where the rate of return on savings is uncertain, in contradistinction to the 'income risk' type, where future income streams are uncertain (Sandmo, 1970).

The total effect of capital risk on consumption can be decomposed into an 'income effect' and a 'substitution effect', the intuition behind each being relatively straightforward. An increase in risk makes the agent less inclined to expose his resources to potential loss and therefore more inclined to increase consumption - the substitution effect. On the other hand, an increase in risk makes it necessary to save more to protect against uncertainty in future wealth - the income effect. It is the substitution effect that is the intuition behind the TOT-volatility channel of the resource curse proposed in Section 1 - increased volatility in the rate of return on savings will cause a decrease in savings, with negative implications for growth.

Sandmo (1970) shows that the substitution effect on consumption is positive if risk aversion is present, while the income effect is negative if absolute risk aversion is decreasing in consumption. (Both conditions are characteristics of our utility function.) The total effect depends on which of these two effects is larger; if the substitution effect is larger (resp. smaller) than the income effect, increased risk will result in a lower (resp. higher) savings rate. The two effects balance if $\gamma = 1$; if $\gamma < 1$, the substitution effect is larger, and increased risk results in a lower savings rate; if $\gamma > 1$, the opposite is true.

The TOT-volatility channel postulated in Section 1 is that resource-price volatility negatively affects economic growth. The model confirms the existence of this channel for $\gamma < 1$. If $\gamma > 1$, TOT volatility is actually beneficial for economic growth, contradicting the predictions of Section 1 and empirical evidence.

So, the 'standard' TOT channels of the resource curse, as predicted in Section 1, both hold if $\gamma < 1$. In this case, a low proportional growth rate of TOT and relatively high volatility in the TOT both negatively affect economic growth. To the extent that these are general characteristics of resource-abundant countries, the resource curse is at play as predicted.

On the other hand, if $\gamma > 1$, the model's predictions are in stark contrast to those of Section 1. In this case, a low proportional growth rate of TOT and high volatility in TOT are actually *beneficial* for economic growth. If these conditions apply in general to resource-abundant countries, the somewhat counterintuitive prediction is that these countries will benefit.

4 Conclusion

While both the level and volatility of the TOT are regularly shown empirically to be significant and robust determinants of economic growth, there is relatively little in the way of research that attempts to characterise these links. As a result, very little is known about how the TOT might play a part in the resource curse, despite this being an oft-cited possibility. This paper has attempted to provide a theoretical grounding for the many empirical analyses that recognise the effects of the TOT on economic growth.

The TOT were endogenised in a stochastic growth model, closed-form solutions of which were derived using the theory of dynamic programming. Under the simplest form of the model, an increase in the average effective rate of return was shown to increase the savings rate if the coefficient of relative risk aversion, γ, is less than unity, and to decrease it if $\gamma > 1$. A mean-preserving increase in the volatility of the effective rate of return was shown to decrease the savings rate if $\gamma < 1$, and to increase it if $\gamma > 1$.

Employing a first-difference filter that approximated the growth rate of consumption, an increase in the average effective rate of return was shown to increase the growth rate of consumption regardless of the value of γ, while an increase in the volatility of the effective rate of return was shown to decrease consumption growth if $\gamma < 2$ and to increase it if $\gamma > 2$.

It was noted, however, that this analysis failed both to explicitly consider the TOT and to extend to growth in output, so that the TOT channel of the resource curse could not be explored. To address this limitation, the model was augmented with further (empirically justified) assumptions. As a result, the model was able to explicitly model the effects of the level and volatility of the TOT on the growth rate of wealth.

It was shown that an increase in the average proportional growth rate of the TOT would decrease savings and the growth rate of wealth if $\gamma < 1$, with the opposite effects if $\gamma > 1$. The case $\gamma < 1$ is thus consistent with the traditional TOT-level channel of the resource curse, where a low growth rate of the TOT of resource-producers chokes their economic growth. The case $\gamma > 1$, on the other hand, yields a result at odds with both intuition and empirical evidence.

An increase in the volatility of the TOT was shown to negatively affect savings and output growth if $\gamma < 1$, with the opposite effects if $\gamma > 1$. Once again, the case $\gamma < 1$ conforms to the traditional TOT-volatility channel of the resource curse, where high volatility in the TOT of resource-abundant countries negatively influences their prospects for economic growth. The case $\gamma > 1$ again yields a result that contradicts both intuition and empirical evidence, viz. that volatility in the TOT is good for both savings and growth.

The contrasting effects of an increase in the growth rate of the TOT on savings for γ less than and greater than unity were shown to be the result of two opposing consumption effects induced by such an increase. An increase in the average rate of return encourages a higher rate of saving to take advantage of the higher return. At the same time, however, the increased expected future wealth encourages higher current consumption, a result of the inclination to smooth consumption. For $\gamma < 1$, the first effect dominates, so that savings and thus output growth increase, and vice-versa if $\gamma > 1$.

Similarly, the contrasting effects of an increase in the volatility of the TOT were shown to be the result of two opposing consumption effects. The first is an inclination to save less, so that a lower level of savings is exposed to the capital risk. The second is an inclination to save more to provide a buffer against potentially low future wealth. For $\gamma < 1$, the first effect dominates, the result of which is a decrease in savings and a decrease in economic growth. The opposite result holds for $\gamma > 1$.

It is important to note that the purpose of this paper has been to characterise the conditions under which the resource curse might operate through the TOT, rather than to consider whether it in fact does or not; that would be the subject of a more empirical analysis.

References

R.J. Barro and X. Sala-i-Martin. *Economic Growth.* McGraw-Hill, New York, 1995.

R. Bellman. The theory of dynamic programming. *Proceedings of the National Academy of Sciences of the United States of America*, 38(8):716–719, 1952.

C. Blattman, J. Hwang, and J.G. Williamson. Winners and losers in the commodity lottery: The impact of terms of trade growth and volatility in the Periphery 1870-1939. *Journal of Development Economics*, 82(1):156–179, 2007.

M. Bleaney and D. Greenaway. The impact of terms of trade and real exchange rate volatility on investment and growth in sub-Saharan Africa. *Journal of Development Economics*, 65(2):491–500, 2001.

J.L. Combes and P. Guillaumont. Commodity price volatility, vulnerability and development. *Development Policy Review*, 20(1):25–39, 2002.

A. Deaton and G. Laroque. On the behaviour of commodity prices. *The Review of Economic Studies*, pages 1–23, 1992.

A. Deaton and R.I. Miller. International Commodity Prices, Macroeconomic Performance and Politics in Sub-Saharan Africa. *Journal of African Economies*, 5(3):99–191, 1996.

A.K. Dixit and R.S. Pindyck. *Investment Under Uncertainty.* Princeton University Press, Princeton, New Jersey, 1994.

W. Easterly, M. Kremer, L. Pritchett, and L.H. Summers. Good Policy or Good Luck? *Journal of Monetary Economics*, 32(3):459–483, 1993.

S. Fischer. The role of macroeconomic factors in growth. *Journal of Monetary Economics*, 32(3):485–512, 1993.

J.A. Frankel. The natural resource curse: a survey. *NBER Working Paper*, 2010.

H. Hotelling. The Economics of Exhaustible Resources. *The Journal of Political Economy*, 39(2):137–175, 1931.

D. Levhari and T.N. Srinivasan. Optimal savings under uncertainty. *The Review of Economic Studies*, 36(2):153–163, 1969.

I.M.D. Little, R.N. Cooper, W.M. Corden, and S. Rajapatirana. *Boom, crisis, and adjustment.* Oxford University Press, 1993.

M. Lutz. The effects of volatility in the terms of trade on output growth: new evidence. *World Development*, 22(12):1959–1975, 1994.

T.R. Malthus. An Essay on the Principle of Population. *J. Johnson*, 1798.

E.G. Mendoza. Terms-of-trade uncertainty and economic growth. *Journal of Development Economics*, 54(2):323–356, 1997.

E.S. Phelps. The accumulation of risky capital: A sequential utility analysis. *Econometrica: Journal of the Econometric Society*, 30(4):729–743, 1962.

R.S. Pindyck. Volatility and commodity price dynamics. *Journal of Futures Markets*, 24(11):1029–1047, 2004.

R. Prebisch. *The economic development of Latin America and its principal problems*. United Nations, New York, 1950.

G. Ramey and V.A. Ramey. Cross-country evidence on the link between volatility and growth. *The American Economic Review*, 85(5):1138–1151, 1995.

J.D. Sachs and A.M. Warner. Natural resource abundance and economic growth. *NBER Working Paper*, 1995.

A. Sandmo. The effect of uncertainty on saving decisions. *The Review of Economic Studies*, 37(3):353–360, 1970.

H.W. Singer. The distribution of gains between investing and borrowing countries. *The American Economic Review*, 40(2):473–485, 1950.

A Proof of Lemma 1

We employ the technique of dynamic programming, as originally developed by Bellman (1952). Let $k_t = A_t/p_t$, wealth at time t in units of importables. The objective is to maximise expected lifetime welfare

$$E\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right], \quad \text{where} \quad u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma},$$

by choosing a consumption plan[4]

$$0 \leqq c_t \equiv f(k_t) \leqq k_t, \quad t = 0, 1, 2, \ldots$$

subject to the intertemporal resource constraint

$$k_{t+1} = r_t(k_t - c_t).$$

The expected lifetime welfare under a defined consumption plan is a function only of initial wealth; define $V(k_0)$ as the expected lifetime welfare attainable given initial wealth k_0:

$$V(k_0) \equiv E\left[\sum_{t=0}^{\infty}\beta^t \frac{c_t^{1-\gamma}}{1-\gamma}\right]. \tag{27}$$

Because the lifetime welfare function is intertemporally additive, we may write

$$\begin{aligned} V(k_0) &= u(c_0) + \beta EV(k_1) \\ &= \frac{c_0^{1-\gamma}}{1-\gamma} + \beta EV\big[r_0(k_0 - c_0)\big]. \end{aligned} \tag{28}$$

If $f(k)$ is an optimal policy, then initial consumption $c_0 = f(k_0)$ must maximise (28) over $0 \leqq c \leqq k_0$, i.e.

$$V(k_0) = \max_{0 \leqq c \leqq k_0}\left[\frac{c^{1-\gamma}}{1-\gamma} + \beta EV\big[r_0(k_0 - c)\big]\right]. \tag{29}$$

Since $u'(c) = c^{-\gamma}$, $u'(0) = \infty$ and we are guaranteed a maximum in the open interval $(0, k_0)$. Differentiating (29) with respect to c and equating the derivative to zero yields

$$c^{-\gamma} = \beta E\left[r_0 V'\{r_0(k_0 - c)\}\right]. \tag{30}$$

Let $c = f(k_0)$ be the solution to (30); substituting it into (28) yields

$$V[k_0] = \frac{\big[f(k_0)\big]^{1-\gamma}}{1-\gamma} + \beta EV\big[r_0(k_0 - f(k_0))\big]. \tag{31}$$

Differentiating both sides of (31) with respect to k_0, we get

$$\begin{aligned} V'[k_0] &= f'(k_0)[f(k_0)]^{-\gamma} + \beta E\left[r_0[1 - f'(k_0)]\, V'[r_0(k_0 - f(k_0))]\right] \\ &= f'(k_0)[f(k_0)]^{-\gamma} + \beta E\left[r_0 V'[r_0(k_0 - f(k_0))]\right] \\ &\qquad - \beta E\left[r_0 f'(k_0)\, V'[r_0(k_0 - f(k_0))]\right], \end{aligned} \tag{32}$$

[4]We may restrict our search to general consumption 'policies' of the form $c_t = f(k_t)$ because the r_t are i.i.d. with *known* distribution, and the discount factor β is constant through time.

where we have exploited the linearity of the E operator. Using (30), (32) may be simplified to

$$V'[k_0] = \big[f(k_0)\big]^{-\gamma}, \tag{33}$$

or alternatively

$$\big[f(k_0)\big]^{-\gamma} = \beta E\left\{r_0\big[f\big(r_0[k_0 - f(k_0)]\big)\big]^{-\gamma}\right\}. \tag{34}$$

Note that since u is concave (so that u' is monotonically decreasing), (33) implies that the optimal policy, where it exists, is unique. (34) is easily recognised as the myopic rule in intertemporal utility maximisation, and as such is the functional equation which has the optimal consumption policy $f(k)$ as its solution. We can therefore drop the time indices and rewrite it as the fundamental equation:

$$\big[f(k)\big]^{-\gamma} = \beta E\left\{r\big[f\big(r[k - f(k)]\big)\big]^{-\gamma}\right\}. \tag{35}$$

This condition, together with the transversality condition

$$\lim_{t\to\infty} E(\beta^t k_t[f(k_t)]^{-\gamma}) = 0, \tag{36}$$

constitute necessary and sufficient conditions for $f(k)$ to be the optimal policy. We now check the *ansatz* $f(k) = \lambda k$, where λ is a constant, against these conditions. Substituting into (35) yields

$$\begin{aligned}\lambda^{-\gamma}k^{-\gamma} &= \beta E\left\{r\big[\lambda r(1-\lambda)k\big]^{-\gamma}\right\}\\ &= \beta\lambda^{-\gamma}k^{-\gamma}(1-\lambda)^{-\gamma}E[r^{1-\gamma}],\end{aligned}$$

so that

$$(1-\lambda)^{\gamma} = \beta E[r^{1-\gamma}]. \tag{37}$$

Assuming $0 < \beta E[r^{1-\gamma}] < 1$, (37) defines λ such that $0 < \lambda < 1$, which is required for feasibility. It remains to check that $f(k) = \lambda k$, with λ defined by (37), satisfies the transversality condition in (36). Given our *ansatz* policy, the intertemporal resource constraint is

$$\begin{aligned}k_t &= (1-\lambda)k_{t-1}r_{t-1}\\ &= (1-\lambda)^2 k_{t-2}r_{t-1}r_{t-2}\\ &\;\vdots\\ &= (1-\lambda)^t k_0 \prod_{\tau=0}^{t-1} r_\tau.\end{aligned} \tag{38}$$

Thus,

$$\begin{aligned}\beta^t k_t[f(k_t)]^{-\gamma} &= \beta^t k_t \lambda^{-\gamma}k_t^{-\gamma}\\ &= \beta^t\lambda^{-\gamma}k_t^{1-\gamma}\\ &= \beta^t\lambda^{-\gamma}(1-\lambda)^{t(1-\gamma)}k_0^{1-\gamma}\prod_{\tau=0}^{t-1} r_\tau^{1-\gamma},\end{aligned} \tag{39}$$

so that

$$E\left[\beta^t k_t [f(k_t)]^{-\gamma}\right] = \beta^t \lambda^{-\gamma}(1-\lambda)^{t(1-\gamma)} k_0^{1-\gamma} E\left[\prod_{\tau=0}^{t-1} r_\tau^{1-\gamma}\right]$$
$$= \beta^t \lambda^{-\gamma}(1-\lambda)^{t(1-\gamma)} k_0^{1-\gamma}[E(r^{1-\gamma})]^t, \qquad (40)$$

with the last equality following from the fact that the r_t are independent and identically distributed. Finally, using (37), we can simplify (40) to

$$E\left[\beta^t k_t [f(k_t)]^{-\gamma}\right] = k_0^{1-\gamma}\lambda^{-\gamma}(1-\lambda)^t, \qquad (41)$$

and since we have assumed $0 < \lambda < 1$, we have that

$$\lim_{t\to\infty} E\left[\beta^t k_t [f(k_t)]^{-\gamma}\right] = 0,$$

so that the transversality condition is satisfied. Thus, $f(k) = \lambda k$, with λ as defined, is the unique optimal policy. Substituting the optimal consumption policy into (3) yields (6).