Industrialisation and surplus labour: A general equilibrium model of sleep, work and leisure

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Abstract

We present a general equilibrium model in which agents allocate their time to sleep, work, eating or a variety of "leisure" activities. We assume that the manufacturing sector which produces the leisure commodities is characterised by increasing returns to scale and monopolistic competition. We show that under certain conditions the economy can be characterised by multiple equilibria. One equilibrium is a high leisure, high work, low sleep equilibrium (the "developed country" equilibrium) while the other is a low leisure, low work, high sleep equilibrium (the "peasant economy" equilibrium). The low development equilibrium is characterised by "surplus labour" in the sense that the same agricultural output can be produced in the high equilibrium with fewer workers. The reason is that the supply of additional varieties of leisure commodities elicits increased work effort by everyone. The low development trap is similar to that in the Murphy, Shleifer and Vishny "big push" model, except that the externalities work through the consumption side.

We consider whether state intervention might be able to move society from the "peasant" economy state to the industrialised one. This requires more than simple coordination of investment (the "big push"), since there is deficient supply of industrial workers. A "poll tax" of the sort imposed by British colonial governments can be effective in supporting the emergence of an industrialised sector. It is, however, likely to lead to a dualistic and distorted economy in which the agricultural population is exploited at the expense of the urban sector.

JEL codes: J2 O1

1 Introduction

Many insights of the classical theories of development have been recast mathematically in the last twenty years. The importance of the connection between industrialisation, increasing returns and coordinated investment decisions, i.e. the "big push", was formalised by Murphy, Shleifer and Vishny (1989b). In a companion piece (1989a) they explore the role of agriculture and income distribution for industrialisation. In similar vein Krugman (1991, 1995) showed how core-periphery relations might emerge and remain stable over time.

One of the key assumptions of "high development theory" – the ubiquitous existence of surplus labour – has not been reexamined in this process of retrieval. However, as Krugman (1995) has noted, the existence of elastic labour supplies is important for many of the "big push" industrialisation accounts. Indeed the original Rosenstein-Rodan (1943) article put surplus labour squarely at the centre of the treatment:

> The assumptions in the case under discussion are: that there exists an "agrarian excess population" in Eastern and South-Eastern Europe amounting to 20-25 million people out of the total population of 100-110 million, i.e., that about 25% of the population is either totally or partially ("disguised unemployment") unemployed.

The magnitude of the presumed surplus labour is large, but seems to have been commonly accepted among development economists of the day. Indeed, the idea that labour is inefficiently utilised in developing economies seems *prima facie* plausible, even today. Travellers from advanced economies frequently chafe at the lack of punctuality that they perceive in these societies.

The initial impetus for the theories of surplus labour was provided by studies of agriculture which suggested that much labour power was "idle" for considerable periods in these economies. Such "idleness" was also commonly remarked upon in the colonial period by colonial administrators and settler cultivators. Some of these claims were undoubtedly motivated by special interests. Collier and Lal (1986), for instance, argue that the owners of Kenyan plantations used such claims to lobby for laws that forced Kenyan smallholders into wage labour at unfavourable rates. Nevertheless it is striking how differences over punctuality and time utilisation seem pervasive in the interactions between individuals raised in different societies.

Indeed changes in the way time is utilised are arguably among the most fundamental processes in economic development. The evolution of increasingly accurate watches and clocks occurred in tandem with the spread of commerce and capitalist enterprise (Landes 2000). The idea that "time is money" has a venerable lineage but it is only since Becker's famous paper on time utilisation (Becker 1965) that modern economists have started to pay more attention to this aspect of economic life[1].

A simple Beckerian model can explain much of the differences in time utilisation between and within societies. In such models the opportunity cost of time is the wage foregone. Consequently the value of time is higher for individuals with higher earnings potential. One would therefore predict that such individuals would substitute activities that are costly in time (such as preparing a meal from scratch) with activities that may have higher input costs but are less costly in time (such as buying prepared meals). If individuals in high income societies are more productive it is easy to see why they might also be more attentive to time, since the opportunity cost to them of "wasted time" (leisure time between activities) is much higher.

Such accounts, however, in themselves cannot sustain multiple equilibria - situations in which the **same** individuals with the same productivity characteristics may choose in one context to supply a lot of labour in order to sustain a high level of consumption, while under different conditions opt for low labour. Below we will present a model in which we graft a Beckerian time allocation model on to a Krugmanesque general equilibrium model with imperfect competition. It turns out that such a model behaves in ways that mimic the Rosenstein-Rodan development trap. In order to pay for the commodities produced by manufacturing, the agricultural workers choose to work harder. If there are sufficiently many goods available this supply response is strong enough to sustain both the demand for the manufactured goods as well as to staff those factories. For an isolated firm, however, this labour supply response may be insufficiently strong, particularly if there are complementarities in consumption. Consequently our model economy may be able to sustain two equilibria: a "peasant" economy, in which everyone is involved in agriculture, sleeps a lot and consumes no manufactured goods; and an "industrial economy" in which the bulk of the labour force works in manufacturing, a much smaller workforce in agriculture supplies the same output, everyone works much harder but also engages in many more "leisure" activities.

In this model, as in the Murphy et al. (1989b) "big push" model one requires a critical mass of industrial firms before the process of industrialisation is sustainable. In our model, however, there is an additional externality associated with industrial production not considered in the Murphy et al. (1989b) paper. The supply of additional varieties of manufactured items may stimulate consumption and hence work effort. It is because variety is sufficiently prized that workers are willing to forego sleep to work and play harder.

The importance of the nexus between consumption and industrialisation has been argued in a number of different contexts. The debates in the Soviet Union in the 1920s about the importance of heavy industry versus the provision of consumer goods was really about this issue. The case of the Kulaks suggests that in the absence of consumer goods the peasant sector is happy to maintain a comfortable level of subsistence and is unwilling to supply extra effort. The failure of the command driven economy to elicit the required work effort suggests that a "big push" that does not provide positive incentives to the workers will not succeed. In the absence of positive incentives the state is tempted to resort to force. Indeed what is striking about many "third world" experiments with "modernisation" is how they were built on coercive foundations. We will sketch out a suggested link between these processes below.

The connection between consumption and industrialisation has been argued even in more benign cases.

[1] In the classical labour theory of value there was clearly a connection between labour time and value.

de Vries (1993), for instance, has suggested that the British industrial revolution was preceded by an "industrious revolution" in which British households redirected their attention from non-market forms of production and free time towards production for the market:

> The industrious revolution argument does not identify idle resources (committed to the market, but unused), it identifies differently deployed resources (not committed to the market) and specifies the conditions under which the household alters its demand patterns and simultaneously alters its offer curves of marketed output and/or labour. (p.108)

This distinction between resources that are otherwise committed but that might become available once the right markets are in place is precisely the sense in which there is "surplus labour" in our model.

If an understanding of the importance of these issues is already present in this literature, what is gained by constructing a mathematical model of this process? In fact the surplus labour literature ran aground because many of the key relationships were unclear. The literature found it difficult to reconcile the existence of surplus labour with a positive wage in agriculture. It could not find convincing rationales for why rational peasants would choose to produce at a point where the marginal product ought to be zero. Our model manages to clarify these issues. There are a number of ancillary benefits of the modelling also. In the first place we show that the variety of commodities that are available may be an important factor in wellbeing. Secondly we suggest that sleep and leisure may be non-trivial development issues. Indeed in order to obtain multiple equilibria we require the elasticity of substitution between sleep and leisure to be sufficiently high. Finally our model suggests why force may be a tempting route for a modernising state.

The plan of the discussion is as follows. In the next section we review some of the literature. In Section 3 we present our model. Agents have utility functions with preferences over "eating", engaging in "leisure" or sleeping. They also need to spend time working to produce the commodities necessary for eating and leisure. On the production side we assume constant returns to scale in agriculture, but increasing returns in industry. We assume that the output market for the industrial products is given by Dixit-Stiglitz monopolistic competition (Dixit and Stiglitz 1977). In Section 4 we show that under certain circumstances there will be two stable equilibria: a "peasant economy" in which there is no industry at all and an industrialised economy which is characterised by fairly high levels of variety. In Section 5 we consider various limitations and objections to the findings. We switch focus in Section 6 to consider whether state intervention might be able to move a society from the "peasant economy" state to the highly industrialised one. In particular we consider the impact of a poll tax, of the sort routinely imposed by the British colonial government. We show that this intervention can, indeed, support the emergence of an industrialised sector. It is, however, likely to lead to a dualistic and distorted economy in which the agricultural population is exploited at the expense of the urban sector. Section 7 concludes.

2 Why surplus labour?

Many of the founders of development economics were of the view that surplus labour was a fact. We have already cited Rosenstein-Rodan to that effect. Leibenstein (1986, p.23) was similarly convinced:

> Is the assertion of an absolute labor surplus true? There are two general bases for the belief in the existence of agricultural underemployment in backward economies: (1) casual observation and (2) statistical comparisons of output between what are sometimes assumed to be roughly comparable areas. ... Nevertheless, despite the rough and ready nature of the methods, the results are not without persuasive power. For it often appears that in some areas the land is cultivated by less than 50 per cent of the labor force used in less developed areas, and yet higher yields are achieved.

Georgescu-Roegen (1960, p.14) put it more strongly: "To regard the notion of overpopulation as a myth is undoubtedly a Marxist residual." Indeed he claims that "[the fact that] there exist countries where the actual marginal productivity of labour is zero for all practical purposes, has been admitted by nearly all students of peasant economies." (1960, p.13) An expert group assembled by the United Nations in 1951, which included W. Arthur Lewis and T.W. Schultz came to the conclusion that it seems "safe to assume that for many regions of India and Pakistan, and for certain parts of the Philippines and Indonesia, the surplus [rural population] cannot be less than the pre-war average for the East European Region" (cited in Kao, Anschel and Eicher 1964, p.130). The definition which this group advanced for surplus labour included a zero marginal product of labour in agriculture. Central to all these accounts was the idea that there was a portion of the agricultural population which could be transferred to the cities without this leading to a drop in agricultural output, even without any changes in the technology utilised or the amount of capital deployed.

Kao et al. (1964, p132) note in their review of this literature that these accounts had to answer three questions:

> First, if labor is unemployed or otherwise wasted, why are techniques not introduced which use less land and capital relative to labor? Second, with given technology (fixed capital-land-labor ratios), why is labor used to the point where no returns are forthcoming? Employers of hired labor lose money when they pay a wage to labor whose product is zero or negligible. The self-employed who produce nothing would do better to hire out their surplus labor for a wage. Third, why are wages higher than the marginal product?

On the first question Eckaus (1955) suggested that the substitutability between labour and the other factors may be more limited than economists generally assume. This problem might be compounded by market imperfections. On the second, Georgescu-Roegen (1960) suggested that the peasant household maximises total output rather than profits. Consequently it will, in fact, employ labour to the point where the marginal product of labour is zero. Implicit in this view, of course, is that the peasant household is not able to sell some of its low productivity labour on the labour market. In short the opportunity cost of that labour is understood to be zero also. On the third question a variety of solutions were suggested in the literature. Leibenstein (1957, 1986) pioneered "nutritional efficiency wage" theories, by suggesting that there might be a link between productivity and the wage. Employers were willing to pay wages above the market clearing level because at lower wages the workers would have insufficient energy in order to produce the output. A different explanation was offered by Lewis (1954), who suggested that peasant households effectively paid their members the average product, rather than the marginal product. This household subsistence level set the "traditional wage" which become the norm governing transactions in other labour markets also. It is, of course, not clear why any agricultural producer would want to pay this wage to outsiders – and in all of these societies there was a spot market for casual labour.

Indeed questions about the validity of the "surplus labour" thesis emerged fairly early. One of the most influential critiques came from Theodore W. Schultz who used the influenza pandemic in India as a "natural experiment" to test this thesis (Ray 1998, pp.358–359). The fact that agricultural output fell during this episode suggested that the marginal product of agricultural labour was not zero. This was not a completely clean test of the thesis, as pointed out by Sen (1967a, 1967b) since land holdings were not redistributed during this period (for a rejoinder see Schultz 1967). Furthermore the pandemic would not have killed completely at random.

Studies in other parts of the world have also cast doubt on the surplus labour thesis (Jorgenson 1981, Kao et al. 1964, Oshima 1958). In particular more careful attention to seasonal fluctuations in work suggested that there might be surplus labour in the slack periods, but that the labour constraint was probably binding at peak times. The review article by Kao et al. (1964, p.141) concludes:

> To date, there is little reliable evidence to support the existence of more than token – 5 per cent – disguised unemployment in underdeveloped countries as defined by a zero marginal product of labor and the condition of *ceteris paribus*.

Given the fact that the surplus labour thesis was always somewhat contentious it is curious that it loomed so large in the early treatments of development economics. The reason for this is probably twofold. In the first instance it palpably looked as though there ought to be surplus labour in agriculture. There seemed to be considerable "slack" in the system. If the marginal product of labour was not zero, it seemed hard to understand why production was not increased, given the evident poverty of these areas. It seemed inconceivable that people would choose to be so poor. Secondly the existence of surplus labour seemd to offer a simple solution to a developmental bottleneck that the early literature identified. Industrial production in the cities required a work force. Furthermore this population had to be fed. Both problems seemed to require efficiency gains in agriculture. In the presence of surplus labour, however, people could be costlessly transferred from the agricultural to the industrial sectors. This model of development was sketched out by Rosenstein-Rodan (1943) and served as the basis of the models by Lewis (1954) and Ranis and Fei (1961).

A less charitable interpretation of the early success of the surplus labour theories is provided by Krugman (1995). He suggests that many of the interesting elements raised in "high development theory": cumulative causation, linkages, coordination failures, the importance of the "big push" were all concepts that depended in one form or another on the critical role of economies of scale. At the time these accounts could not be modelled – which became a drawback as the economics profession became more mathematical.

> The exception proves the rule. Lewis's surplus labor concept was the model that launched a thousand papers; even though surplus labor assumptions were already standard among development theorists, the empirical basis for assuming surplus labor was weak, and the idea of external economies/strategic complementarity was surely more interesting. The point was, of course, that precisely because he did not mix economies of scale into his framework, Lewis offered theorists something they could model using available tools. (Krugman 1995, p.28)

In this paper we will reconnect the surplus labour concept to the question of economies of scale. We will suggest that there may be situations in which something like surplus labour exists. Nevertheless the marginal product of agricultural labour is never zero. The poverty trap exists because of the lack of key markets – notably for consumption items. The peasants in our model are not choosing a life of "leisure" over a life with a higher standard of living. The latter is simply not available to them.

The benefit of developing the model is not simply to give a veneer of mathematical respectability to accounts that suggest that effort by the peasants could be increased in some situations. It also suggests a reason why many colonial and "modernising" states have resorted to coercive measures to elicit that effort.

3 A general equilibrium model of time allocation

The overall structure of the model is rather simple, although the details might obscure this. We assume that all agents are identical in their preferences and productivity. Furthermore we will allow mobility between agriculture and industry, so that wages in both sectors will be identical. The only difference will be in the technology of production and hence market structure. Agriculture we assume is characterised by constant returns to scale production and perfect competition, while the varieties of industrial goods are produced by means of an increasing returns technology under conditions of monopolistic competition. Free entry of firms will erode profits to zero, however. With no profits being made our only concern will be to ensure that all product and factor markets clear. Since we model production with only one factor, it is the labour market that needs to clear. Equilibrium requires both individual labour supply to be optimal given the consumers' preferences as well as a distribution of the workforce between agriculture and industry that ensures that the required output can be produced. In short the individual's optimal time allocation as well as the "urban-rural" balance become endogenous in this model.

3.1 Consumption

We assume that individuals have a common utility function over activities given by

$$U = Z_A^{\gamma} \left(Z_M^{\rho} + T_S^{\rho}\right)^{\frac{1-\gamma}{\rho}} \tag{1}$$

where $\rho < 1$, Z_A is the activity of consuming the agricultural good , T_S is sleeping time and Z_M is an aggregate of n different "leisure" activities, involving the consumption of manufactured goods which is given by

$$Z_M = \left(\sum_{i=1}^{n} z_{mi}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{2}$$

with $\sigma > 1$ so that it is possible for there to be zero consumption of some of these activities. This particular utility function implies that "sleeping" and the different varieties of leisure activities are thought of as a bigger composite commodity ("recreation") which is traded off against "eating". This nested form is chosen to allow for non-unit elasticities of substitution between leisure and sleep while keeping the overall problem fairly tractable[2]. The parameter σ is the elasticity of substitution between different varieties of leisure. The elasticity of substitution between sleeping and "leisure" (in aggregate) is given by $\frac{1}{1-\rho}$.

We make the Beckerian assumptions that each activity Z_A, $z_{m1} \ldots , z_{mn}$ is produced through a constant coefficient production function involving time and commodities such that

$$x_A = a_A Z_A, \qquad T_A = b_A Z_A \tag{3a}$$

$$x_{mi} = a_{mi} z_{mi}, \qquad T_{mi} = b_{mi} z_{mi} \tag{3b}$$

The money budget constraint can be written as

$$x_A + p_1 x_{m1} + \ldots + p_n x_{mn} \leq w \left(T^* - T_A - \sum_{i=1}^{n} T_{mi} - T_S \right) + I \tag{4}$$

where T^* is the total endowment of time, w is the wage rate and I is non-wage income. We have used the price of the agricultural commodity as the numeraire.

Substituting equations 3a and 3b into the budget constraint we get, in the standard way, the full budget constraint in terms of the time available

$$\left(a_A + w b_A\right) Z_A + \sum_{i=1}^{n} \left(p_i a_{mi} + w b_{mi}\right) z_{mi} + w T_S \leq w T^* + I$$

We write this as

$$\pi_A Z_A + \sum_{i=1}^{n} \pi_{mi} z_{mi} + w T_S \leq w T^* + I \tag{5}$$

where π_A and π_{mi} are the "full prices" of the activities Z_A and z_{mi}.

Because the utility function has a nested structure where each subutility function is homogeneous of degree zero, it turns out that doing the optimisation is equivalent to doing the optimisation in stages (Gorman 1959, Deaton and Muellbauer 1980, Chapter 5). In the first stage the individual optimises a Cobb-Douglas utility function involving preferences over consuming the basic necessities (the agricultural commodity) and a composite utiltiy function comprising of leisure and sleeping time. This yields the demand

$$Z_A = \gamma \frac{w T^* + I}{\pi_A} \tag{6}$$

We can derive the balance of the demands in similar stage-wise fashion, provided that we define an appropriate price index for the composite good Z_M. It transpires that the price index π_M for the composite good Z_M corresponding to the full budget constraint (equation 5) will be given by

$$\pi_M = \left(\sum_{i=1}^{n} \pi_{mi}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{7}$$

The demand functions for the composite "leisure" activity and sleeping time, expressed in terms of this price index are:

$$Z_M = (1-\gamma) \frac{\pi_M^{r-1} \left(w T^* + I\right)}{\left(\pi_M^r + w^r\right)} \tag{8a}$$

$$T_S = (1-\gamma) \frac{w^{r-1} \left(w T^* + I\right)}{\left(\pi_M^r + w^r\right)} \tag{8b}$$

[2] With unit elasticities constant shares of the overall time budget are spent on sleep – i.e. sleep no longer varies!

where $r = \frac{\rho}{\rho-1}$. The demands for the individual varieties of leisure activities will be given by

$$z_{mi} = \left(\frac{\pi_M}{\pi_{mi}}\right)^{\sigma} Z_M \tag{9}$$

3.1.1 The demand for sleep

Some implications can be derived immediately from the demand functions. Firstly, it is evident that

$$\frac{\partial T_S}{\partial \pi_M} = -\left(1-\gamma\right)\frac{rw^{r-1}\pi_M^{r-1}\left(wT^* + I\right)}{\left(\pi_M^r + w^r\right)^2}$$

The sign of this will depend on the sign of r. If $0 < \rho < 1$ then $\frac{\partial T_S}{\partial \pi_M} > 0$. This case corresponds to an elasticity of substitution between leisure as a group and sleep in excess of one. As the price of leisure increases the individual substitutes towards sleep in the expected way. More paradoxical is the case where $\rho < 0$. In this case $\frac{\partial T_S}{\partial \pi_M} < 0$, so an increase in the price of leisure actually leads to a decrease in sleep. The reason for this is that the price increase also has an income effect. Since the inividual is not substituting away from the higher priced activity sufficiently, the consumption of the **other** activities also decreases.

Secondly, a consideration of equation 7 shows that introducing additional varieties of leisure activities will have the effect of *lowering* the price index π_M. This follows as $\sigma > 1$ and hence we can write the price index in the form

$$\pi_M = \frac{1}{\left(\sum_{i=1}^{n}\frac{1}{\pi_{mi}^{\sigma-1}}\right)^{\frac{1}{\sigma-1}}}$$

This point is critical for understanding the behaviour of the model. The *availability* of additional varieties acts analogously to a decrease in price of the aggregate. The impact of this "price decrease" will depend again on the sign of r. With $0 < \rho < 1$ the increase in variety will lead to a decrease in sleep time, while with $\rho < 0$ it will lead to an increase. In short

$$\frac{\partial T_S}{\partial n} \lesseqqgtr 0 \text{ as } \rho \gtreqqless 0$$

The second possibility is again due to an income effect. In this case the ability to consume more leisure means that the individual is better off. However the individual is not substituting towards leisure sufficiently rapidly and hence takes some of this additional income in the form of increased sleep.

Calculating the impact of a wage increase is complicated by the fact that w feeds into the "prices" π_A and π_M. Indeed

$$\begin{aligned}
\frac{\partial \pi_A}{\partial w} &= b_A \\
\frac{\partial \pi_M}{\partial w} &= \left(\sum_{i=1}^{n}\pi_{mi}^{1-\sigma}\right)^{\frac{\sigma}{1-\sigma}}\left(\sum_{i=1}^{n}\pi_{mi}^{-\sigma}b_{mi}\right) \\
&= \pi_M\sum_{i=1}^{n}\frac{\pi_{mi}^{1-\sigma}}{\left(\sum_{i=1}^{n}\pi_{mi}^{1-\sigma}\right)}\frac{b_{mi}}{\pi_{mi}}
\end{aligned}$$

In this form it is clear that the relative change in π_M induced by a change in w is a weighted average of proportions of the labour times b_{mi} in the total prices π_{mi}. The weights are $\frac{1}{\pi_{mi}^{\sigma-1}}$, i.e. activities with a low price will have a larger weighting.

The total impact of a wage increase is given by

$$\frac{dT_S}{dw} = \frac{\partial T_S}{\partial w} + \frac{\partial T_S}{\partial \pi_M}\frac{\partial \pi_M}{\partial w} + \frac{\partial T_S}{\partial \pi_A}\frac{\partial \pi_A}{\partial w}$$

With our assumption of Cobb-Douglas preferences at the first stage, we have $\frac{\partial T_S}{\partial \pi_A} = 0$, although this need not be true in general. This complicated expression is due to the fact that the wage is the opportunity cost of sleep and hence one needs to consider the income and substitution effects induced by changes in the "price" of sleep. However, one also needs to consider changes in the opportunity costs of all other activities. Considering the first term, we have

$$\frac{\partial T_S}{\partial w} = (1-\gamma)\frac{rw^{r-2}\pi_M^r\left(wT^* + I\right) - w^{r-2}I\left(\pi_M^r + w^r\right)}{\left(\pi_M^r + w^r\right)^2}$$

This is unambiguously negative if $r < 0$, i.e. if $0 < \rho < 1$. It will be positive if $r > \left(\frac{I}{wT^*+I}\right)\left(1 + \frac{w^r}{\pi_M^r}\right)$. In particular if $I = 0$, it will be positive if $r > 0$. The total derivative is given by

$$\begin{aligned}\frac{dT_S}{dw} &= (1-\gamma)\frac{rw^{r-2}\left(\pi_M^r\right)\left(wT^* + I\right) - w^{r-2}I\left(\pi_M^r + w^r\right)}{\left(\pi_M^r + w^r\right)^2} - \\ &\quad (1-\gamma)\frac{rw^{r-1}\pi_M^{r-1}\left(wT^* + I\right)}{\left(\pi_M^r + w^r\right)^2}\pi_M\sum_{i=1}^{n}\frac{\pi_{mi}^{1-\sigma}}{\left(\sum_{i=1}^{n}\pi_{mi}^{1-\sigma}\right)}\frac{b_{mi}}{\pi_{mi}}\end{aligned}$$

Let $\sum_{i=1}^{n}\frac{\pi_{mi}^{1-\sigma}}{\left(\sum_{i=1}^{n}\pi_{mi}^{1-\sigma}\right)}\frac{b_{mi}}{\pi_{mi}} = \delta$, then

$$\frac{dT_S}{dw} = (1-\gamma)\left[\frac{rw^{r-2}\left(wT^* + I\right)\left(\pi_M^r\left(1 - w\delta\right)\right)}{\left(\pi_M^r + w^r\right)^2} - \frac{w^{r-2}I}{\left(\pi_M^r + w^r\right)}\right]$$

Now $w\delta = \sum_{i=1}^{n}\frac{\pi_{mi}^{1-\sigma}}{\left(\sum_{i=1}^{n}\pi_{mi}^{1-\sigma}\right)}\frac{wb_{mi}}{\pi_{mi}} < 1$ so the sign of the first term in the square brackets depends only on r. In short the total derivative $\frac{dT_S}{dw}$ will have the same sign as the partial derivative $\frac{\partial T_S}{\partial w}$. The term $(1 - w\delta)$ indicates why this should be the case: the effect of the wage increase on the price of leisure is smaller than on the price of sleep, since the cost of time is not the only element in the price index.

3.1.2 Labour supply

It will be of some interest to derive the time spent working in this model. We have

$$\begin{aligned}T_w &= T^* - T_A - \sum T_{mi} - T_S \\ &= T^* - \gamma b_A\frac{wT^* + I}{\pi_A} - (1-\gamma)\frac{\left(wT^* + I\right)}{\left(\pi_M^r + w^r\right)}\left(\sum_{i=1}^{n} b_{mi}\left(\frac{\pi_M}{\pi_{mi}}\right)^{\sigma}\pi_M^{r-1} + w^{r-1}\right) \qquad (10)\end{aligned}$$

Below we will be interested in imposing a zero profit condition. Consequently we set exogenous income I to zero. Furthermore we normalise the time endowment T^* to one. Substituting these in we get (after simplification)

$$T_w = \gamma\frac{a_A}{\pi_A} + (1-\gamma)\frac{\pi_M^{r-1+\sigma}\left(\pi_M^{1-\sigma} - w\left(\sum_{i=1}^{n} b_{mi}\pi_{mi}^{-\sigma}\right)\right)}{\pi_M^r + w^r}$$

observing that $\pi_M^{1-\sigma} = \sum_{i=1}^{n}\pi_{mi}^{1-\sigma}$ we can write this as

$$T_w = \gamma\frac{a_A}{\pi_A} + (1-\gamma)\frac{\pi_M^{r-1+\sigma}\left(\sum_{i=1}^{n}\pi_{mi}^{-\sigma}p_i a_{mi}\right)}{\pi_M^r + w^r} \qquad (11)$$

3.1.3 Some simplifications

What makes this model quite complicated is the fact that different individuals will be facing different prices, depending on their particular opportunity cost of time. In order to simplify matters somewhat, we make the simplifying assumption that the time cost of all types of leisure is the same, i.e.

$$b_{mi} = b_m \qquad (12)$$

We can simplify things further by choosing units so that $a_A = 1$ and each $a_{mi} = 1$. With these normalisations we have

$$\begin{aligned}\pi_A &= 1 + wb_A \\ \pi_{mi} &= p_i + wb_m\end{aligned}$$

where w is the wage of the individual. In general this might be different for different types of individuals. The model rapidly becomes intractable, hence our decision to allow only one wage rate.

In the case where the prices of the manufactured goods p_i all happen to be equal (we will show below that this will be the case in our model), it follows that the full price of any leisure activity π_{mi} will be a constant π_m for any type of individual and

$$\begin{aligned}\pi_M &= \left(n\pi_m^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \\ &= n^{\frac{1}{1-\sigma}}\pi_m \qquad (13)\end{aligned}$$

In this form it is again clear that π_M is decreasing in n. Substituting this expression into equation 11 and simplifying we get:

$$T_w = \gamma\frac{a_A}{\pi_A} + (1-\gamma)\frac{n^{\frac{r}{1-\sigma}}\pi_m^{r-1}p}{n^{\frac{r}{1-\sigma}}\pi_m^r + w^r}$$

where p is the common price of the manufactured goods. It follows that in this particular case, work time either increases or decreases with the number of varieties, depending on the sign of r, i.e. $\frac{\partial T_w}{\partial n} \lesseqgtr 0$ as $r \gtreqless 0$.

3.2 Production and Market Equilibrium

We assume that there are only two types of workers in our economy, agricultural workers and manufacturing workers, with mobility between sectors. Consequently the wage rate will be the same, i.e. $w_A = w_M$. We denote the common wage rate as w. Since all individuals earn the same wage, the "full" prices faced by them will all be equal. This is clearly a simplifying assumption, but one that makes the model more tractable.

The total labour supply is L and we assume that the number of agricultural workers $L_A = \phi L$, i.e. the number of manufacturing workers is $(1-\phi)L$. It is clear that ϕ will be endogenous in this model.

3.2.1 The agricultural good

We assume that the agricultural good is produced under constant returns to scale conditions with input only from labour, i.e.

$$L_A T_{wA} = \psi X_A \qquad (14)$$

where X_A is total market supply and T_{wA} is the time spent working by a farmer. Note that ψ is an inverse measure of the efficiency of agricultural production. The larger ψ, the smaller the marginal product of an agricultural worker.

We know that individual demand is given by

$$\begin{aligned}x_A &= a_A Z_A \\ &= Z_A\end{aligned}$$

It follows that market demand is

$$X_A = L\gamma\frac{w}{1+wb_A}$$

From equation 11 we know that

$$T_{wA} = \gamma\frac{a_A}{\pi_A} + (1-\gamma)\frac{\pi_M^{r-1+\sigma}\left(\sum_{i=1}^n \pi_{mi}^{-\sigma}p_i a_{mi}\right)}{\pi_M^r + w^r}$$

Setting $a_A = 1$ and $a_{mi} = 1$ we get

$$\phi L \left[\gamma \frac{1}{1+wb_A} + (1-\gamma) \frac{\pi_M^{r-1+\sigma} \left(\sum_{i=1}^n \pi_{mi}^{-\sigma} p_i \right)}{\pi_M^r + w^r} \right] = \psi L \gamma \frac{w}{1+wb_A}$$

Finally imposing a zero profit condition on agricultural production implies that marginal cost equals price, i.e. $\psi w = p_A = 1$. Consequently

$$\phi (1-\gamma) \frac{\pi_M^{r-1+\sigma} \left(\sum_{i=1}^n \pi_{mi}^{-\sigma} p_i \right)}{\pi_M^r + w^r} = (1-\phi) \gamma \frac{1}{1+wb_A} \tag{15}$$

This equation is the condition for market equilibrium in the agricultural sector. It has a natural interpretation. The left hand side is the demand for manufacturing goods that has to be funded with wages earned in the agricultural sector, while the right hand side is the payments received from manufacturing workers for agricultural goods consumed.

3.2.2 The manufactured goods

In the case of manufacturing we assume that the work force required to produce output X_{mi} is given by

$$L_{mi} T_{wm} = \alpha + \beta X_{mi} \tag{16}$$

where T_{wm} is the time spent working by a manufacturing worker. This implies that the employer incurs a fixed cost of production αw and then a marginal cost of βw.

Each variety of the manufactured good is produced by a single producer, i.e. the structure of production is Dixit-Stiglitz monopolistic competition (Dixit and Stiglitz 1977). The mark-up of each producer is given in the normal way as

$$p_i = \frac{1}{1 - \frac{1}{|\varepsilon_i|}} MC \tag{17}$$

where $|\varepsilon|$ is the elasticity of demand. In this case it is given by

$$\begin{aligned} \left| \frac{\partial x_{mi}}{\partial p_i} \frac{p_i}{x_{mi}} \right| &= \left| \frac{\partial x_{mi}}{\partial \pi_{mi}} \frac{\partial \pi_{mi}}{\partial p_i} \frac{p_i}{\pi_{mi}} \frac{\pi_{mi}}{x_{mi}} \right| \\ &= \sigma El_{\pi_{mi}, p_i} \end{aligned}$$

where we have used the standard approximation in these models, i.e. we ignore the own-price effect (which is of the order of $\frac{1}{n}$)[3]. Since all consumers face identical "full prices" and budget constraints, the elasticity of aggregate demand with respect to prices will be identical to the elasticity of individual demand.

Now $\pi_{mi} = p_i + wb_m$, so $El_{\pi_{mi}, p_i} = \frac{p_i}{\pi_{mi}}$. Furthermore $MC = \beta w$. Substituting this into the mark-up equation we get

$$p_i = \frac{\sigma \beta + b_m}{\sigma - 1} w \tag{18}$$

Note that the assumption of identical wage rates and identical time cost (equation 12) are vital in allowing us to derive this relatively simple expression. It follows from this equation that all varieties will be priced identically, with the common price p given by equation 18. It is useful to note that as in similar models (Krugman 1991, Krugman 1995) σ functions as an inverse indicator of monopoly power. Values of σ close to one lead to a high mark-up, suggesting that increasing returns are a fairly powerful force in the economy.

Given that all leisure commodities are commonly priced, the full price will be given by

$$\begin{aligned} \pi_{mi} &= \frac{\sigma \beta + b_m}{\sigma - 1} w + b_m w \\ &= \frac{\sigma (\beta + b_m)}{\sigma - 1} w \end{aligned} \tag{19}$$

[3] We will look at the specific case of monopoly production (where we can't ignore this effect) below.

We could also write this as

$$\pi_{mi} = \frac{\sigma\left(\beta + b_m\right)}{\psi\left(\sigma - 1\right)}$$

It follows further (from equation 13) that

$$\pi_M = n^{\frac{1}{1-\sigma}} \frac{\sigma\left(\beta + b_m\right)}{\sigma - 1} w \tag{20}$$

3.2.3 The number of varieties of manufactured goods

With free entry profits will be driven to zero in the manufacturing sector, i.e.

$$\begin{aligned} p_i X_{mi} &= \left(\alpha + \beta X_{mi}\right) w \\ X_{mi} &= \frac{\alpha\left(\sigma - 1\right)}{\beta + b_m} \end{aligned} \tag{21}$$

It follows that the labour demand for manufacturing workers will be given by

$$\begin{aligned} L_M T_{wm} &= \sum_{i=1}^{n} \left(\alpha + \beta \frac{\alpha\left(\sigma - 1\right)}{\beta + b_m}\right) \\ &= \sum_{i=1}^{n} \frac{\alpha\beta\sigma + \alpha b_m}{\beta + b_m} \\ &= \frac{n\alpha\left(\beta\sigma + b_m\right)}{\beta + b_m} \end{aligned}$$

Substituting in the supply of time, from equation 11 this condition becomes

$$\left(1 - \phi\right) L\gamma \frac{1}{\left(1 + wb_A\right)} + \left(1 - \phi\right) L \left(1 - \gamma\right) \frac{\pi_M^{r-1+\sigma} \left(\sum_{i=1}^{n} \pi_{mi}^{-\sigma} p\right)}{\pi_M^r + w^r} = \frac{n\alpha\left(\beta\sigma + b_m\right)}{\beta + b_m} \tag{22}$$

Substituting in the condition for equilibrium in the agricultural goods market (equation 15) we get

$$\begin{aligned} \left(1 - \phi\right) L\gamma \frac{1}{\left(1 + wb_A\right)} + \frac{\left(1 - \phi\right)}{\phi} L \left(1 - \phi\right) \gamma \frac{1}{1 + wb_A} &= \frac{n\alpha\left(\beta\sigma + b_m\right)}{\beta + b_m} \\ n = \frac{\left(1 - \phi\right) L\gamma}{\phi} \frac{\beta + b_m}{\alpha\left(\beta\sigma + b_m\right)} \frac{\psi}{\psi + b_A} & \end{aligned} \tag{23}$$

Note that we have ignored the fact that the quantity of varieties has to be an integer amount, i.e. we have assumed that the zero profit condition can hold exactly. This is an idealisation which will not, however, distort the overall conclusions derived from the model. The interpretation of equation 23 is fairly straightforward. The number of leisure activities that can be sustained in equilibrium increases with the proportion of the population that is engaged in manufacturing. This should be fairly obvious, since the supply of the associated goods depends on manufacturing. The activities also go up with population. This is due to the phenomenon of increasing returns to scale in production. Each production process needs to reach the fixed cost threshold of α before production can commence. So the larger the market, the greater the number of varieties. Indeed a decrease in α will also lead to an increase in n.

We observe that

$$\frac{\partial n}{\partial \beta} = -b_m \frac{\sigma - 1}{\left(\beta\sigma + b_m\right)^2} < 0$$

so a decrease in the efficiency of manufacturing labour (increase in β) will lead to a decrease in the number of varieties. An increase in σ decreases the total number of varieties. This result is due to the interaction of two processes: on the one hand an increase in σ leads to a drop in the price of manufactured goods, which in turn leads to a strong increase in demand; on the other hand, with much lower prices the profit margins are also reduced and it is consequently much harder to reach the break even point, given that the fixed costs have to be met first. An increase in b_m, the leisure time required to engage in the activitiy acts in the opposite way - it effectively increases the monopoly power of the producer and thus enables the threshold to be more easily reached.

The last term on the right hand side of equation 23 is the reciprocal of the full price of the agricultural activity. It serves here as an index for demand in the economy as a whole. The lower the cost the greater the demand and hence the greater the number of varieties that can be sustained in equilibrium.

3.2.4 Equilibrium demand for manufactured goods

Finally we need to check for equilibrium in the manufactured goods market. We have (by our normalising assumption) $x_{mi} = z_{mi}$, but

$$\begin{aligned} z_{mi} &= \left(\frac{n^{\frac{1}{1-\sigma}} \pi_{mi}}{\pi_{mi}} \right)^{\sigma} Z_M \\ &= n^{\frac{\sigma}{1-\sigma}} Z_M \end{aligned}$$

and

$$Z_M = (1-\gamma) \frac{\pi_M^{r-1} w}{\pi_M^r + w^r}$$

Substituting in for the equilibrium value of π_M from equation 20 we get that the individual demand for manufactured good i will be given by

$$x_{mi} = (1-\gamma) \frac{n^{\frac{r}{1-\sigma}-1} \left[\frac{\sigma(\beta+b_m)}{\sigma-1} \right]^{r-1}}{n^{\frac{r}{1-\sigma}} \left[\frac{\sigma(\beta+b_m)}{\sigma-1} \right]^{r} + 1}$$

The market demand for the good is therefore

$$X_{mi} = (1-\gamma) L \frac{n^{\frac{r}{1-\sigma}-1} \left[\frac{\sigma(\beta+b_m)}{\sigma-1} \right]^{r-1}}{n^{\frac{r}{1-\sigma}} \left[\frac{\sigma(\beta+b_m)}{\sigma-1} \right]^{r} + 1} \tag{24}$$

Equating this to the equilibrium supply given in equation 21 we get the condition

$$\begin{aligned} \frac{\alpha(\sigma-1)}{\beta+b_m} &= (1-\gamma) L \frac{n^{\frac{r}{1-\sigma}-1} \left[\frac{\sigma(\beta+b_m)}{\sigma-1} \right]^{r-1}}{n^{\frac{r}{1-\sigma}} \left[\frac{\sigma(\beta+b_m)}{\sigma-1} \right]^{r} + 1} \\ \frac{\alpha}{\left(\frac{(1-\gamma)L}{\sigma n} - \alpha \right)} &= \frac{n^{\frac{r}{1-\sigma}} \sigma^r (\beta+b_m)^r}{(\sigma-1)^r} \end{aligned} \tag{25}$$

Note that this gives valid solutions only if $\frac{(1-\gamma)L}{\sigma n} \geq \alpha$.

3.2.5 The size of the agricultural workforce

Furthermore

$$\frac{L}{n} = \frac{\phi\alpha(\beta\sigma+b_m)(\psi+b_A)}{\gamma(1-\phi)(\beta+b_m)\psi}$$

by equation 23. Substituting in we can rewrite the equilibrium condition as

$$1 = \left(\frac{\phi}{(1-\phi)} \frac{(1-\gamma)}{\sigma\gamma} \frac{(\beta\sigma + b_m)(\psi + b_A)}{(\beta + b_m)\psi} - 1 \right) \left[\frac{\phi}{(1-\phi) L\gamma} \frac{\alpha(\beta\sigma + b_m)}{\beta + b_m} \frac{\psi + b_A}{\psi} \right]^{\frac{r}{\sigma-1}} \frac{\sigma^r (\beta + b_m)^r}{(\sigma - 1)^r} \qquad (26)$$

This equation implicitly defines ϕ as a function of the other parameters. If $r > 0$ then this clearly has a unique solution. Let $G(\phi)$ be the expression on the right hand side. Then $G(0) < 1$. Furthermore $\lim_{\phi \to 1} G(\phi) = \infty$. However, if $r < 0$ then there may be more than one solution! Intuitively the introduction of another variety of leisure activity increases the time spent working so that it may more than pay for itself!

4 An example of general equilibrium with surplus labour

In order to get multiple equilibria we *require* $r < 0$, i.e. $0 < \rho < 1$. This implies an elasticity of substitution between sleep and leisure activities in excess of one. This in turn implies that the introduction of additional varieties (which acts like a decrease in price) elicits a substitution away from sleep and towards leisure. Furthermore as we observed in section 3.1.3 this additional leisure would have to be paid for through an increase in work. In such a world the quest for novelty erodes sleep from two ends: the direct effect of needing to fit in additional activities as well as the work time necessary to pay for the equipment used.

We will show that under these circumstances multiple equilibria are possible. For instance, setting $\rho = \frac{1}{2}$ and $\sigma = 1.5$ and solving for ϕ in equation 26 we will get a quadratic equation which will have two solutions provided that

$$L > \frac{3\alpha\sqrt{3(\beta + b_m)}}{1 - \gamma} \qquad (27)$$

Intuitively we require the total labour force to be large relative to the start up costs (α) and the costs incurred in producing (β) and consuming (b_m) the manufactured goods. The larger the budget share $(1 - \gamma)$ of "recreation", i.e. sleep and leisure, the easier it will be to meet this condition also.

The equilibrium agricultural workforce will be given by

$$\phi = \frac{3L\gamma\psi(\beta + b_m)}{3L\gamma\psi(\beta + b_m) + (1.5\beta + b_m)(\psi + b_A)\left((1-\gamma) L \pm \sqrt{(1-\gamma)^2 L^2 - 27(\beta + b_m)\alpha^2} \right)}$$

We have graphed these values for particular choices of the parameters in Figure 1. It is clear that the same underlying parameters can give rise to vastly different economic outcomes. It is easy to verify that ϕ increases with ψ, so as the productivity of agricultural labour falls a larger proportion of the overall workforce will be required in agriculture regardless of which equilibrium obtains. An increase in the time cost of consuming the agricultural product b_A serves to reduce demand which leads to a smaller agricultural workforce.

An increase in L has the effect of widening the gap between the two equilibria. We show this graphically in Figure 2. We note that there is a lower limit to the proportion of workers in agriculture which is given by the fact that there will always be a demand for agricultural produce (indeed with Cobb Douglas preferences this will be a fixed demand given by $\frac{\gamma L}{\psi + b_A}$) and with the productivity parameter set at a fixed value the work force cannot fall below the number of workers required to produce this output working around the clock. That physical minimum in this case would be given by 0.1. Since the workers value sleep (to some extent) and also require time for eating and consuming leisure goods, the lower limit has to be above this – in the example it is at 0.167.

Given the solutions for ϕ, we can solve for the number of varieties that will be sold in equilibrium using equation 23. In this case

$$n = \frac{(1-\gamma) L \pm \sqrt{(1-\gamma)^2 L^2 - 27(\beta + b_m)\alpha^2}}{3\alpha}$$

We observe that in the small agricultural workforce equilibrium there will be a high number of varieties. On this branch n increases with L. The reverse is true in the large agricultural workforce ("peasant economy")

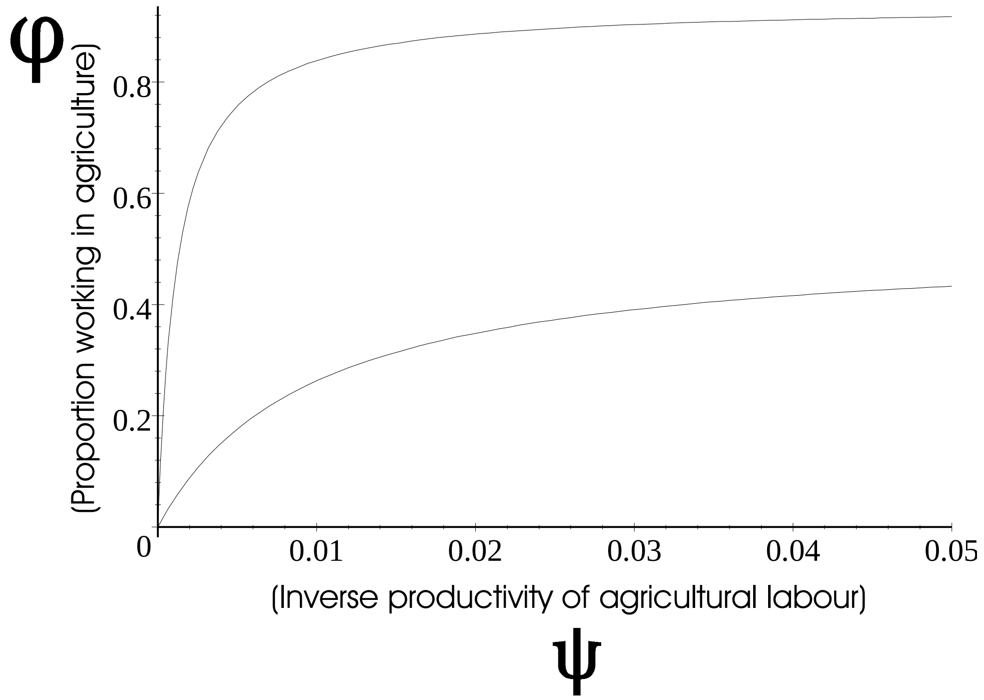

Figure 1: The same parameter values can lead to very different distributions of the workforce between agriculture and industry. Parameter settings: $L = 750$, $\gamma = \frac{1}{2}$, $\alpha = 5$, $\beta = 50$, $b_m = 0.1$ and $b_A = 0.02$.

Figure 2: An increase in the labour force drives the two equilibria further apart. Parameter settings: $\alpha = 5$, $\beta = 50$, $b_m = 0.1$, $\gamma = 0.5$, $\psi = 0.005$, $b_A = 0.02$

	High	Low
n	46.788	3.2124
ϕ	.17619	.75699
w	200	200
p_i	30060	30060
X_A	150000	150000
T_s	3.2124×10^{-2}	.46788
T_w	.56756	.1321
T_m	3.1129×10^{-4}	2.1373×10^{-5}

Parameter settings: $L = 750$, $\alpha = 5$, $\beta = 50$, $b_m = 0.1$, $\gamma = 0.5$, $\psi = 0.005$, $b_A = 0.02$

Table 1: Two very different equilibria arising from the same economic structure

equilibrium. This may seem rather strange, but we will show below that the high ϕ locus is, in fact, an unstable equilibrium which separates the purely agricultural economy from the industrialised one. In Figure 2 we have shown this by means of the arrows which indicate the direction of movement within this space. An increase in L therefore has the unambiguous effect of increasing the basin of attraction for the industrialised equilibrium. The reason for this is increasing returns to scale in industrial production. With a larger workforce there is also a larger market so it is easier for production to achieve the minimum scale at which the fixed costs can be covered. It is obvious that in this (autarkic) world some economies can never industrialise.

With the values for ϕ and n and the given parameter values we can solve for the other variables in the model. A summary of some key quantitities is given in Table 1. It is clear that the high variety, low agricultural work force economy produces *precisely* the same total agricultural output as the low variety economy. This is the sense in which this economy exhibits "surplus labour". Workers can be removed from agriculture without reducing the overall output. The surplus labour in the "peasant economy" equilibrium is occupied in sleep!

Several additional facts are important about this example:

- Agricultural workers are paid their marginal product. Indeed since marginal products are not decreasing in the number of workers, the wage rate is precisely the same in the two economies.
- All markets clear
- Prices in the two economies are identical! The wage rates are equal and the price of each variety is priced at exactly the same mark-up over wages.
- In the high-variety economy people spend more time working, less time sleeping and more time on leisure (about 15 times as much). The last set of numbers are tiny due to the particular parameter values chosen, but this could be different with different settings.
- Incomes are higher in the high-variety economy, because people work longer hours. Utility is higher in this economy because the optimal consumption bundle of the low-variety economy is available but no-one chooses it.

What is driving these patterns? A simple way to get a fix on the underlying dynamics is to focus on the relationships between equations 21 and 24. The former is the break even condition under monopolistic competition and the second is the actual demand for any one of the varieties, given the assumptions about the pricing behaviour of the firms. The two equations have been plotted in Figure 3.

The first point to note is that if *fewer* varieties than the first equilibrium (at. n_1) are in the market, then every manufacturer makes negative profits. From this point on, however, each additional entrant will make positive profits until the high equilibrium is reached (at n_2). The "low-level" equilibrium given in Table 1 is therefore unstable. It is, in fact, the threshold at which industrialisation takes off. The second point is that the shape of the market demand curve is rather curious. Initially each additional variety increases the

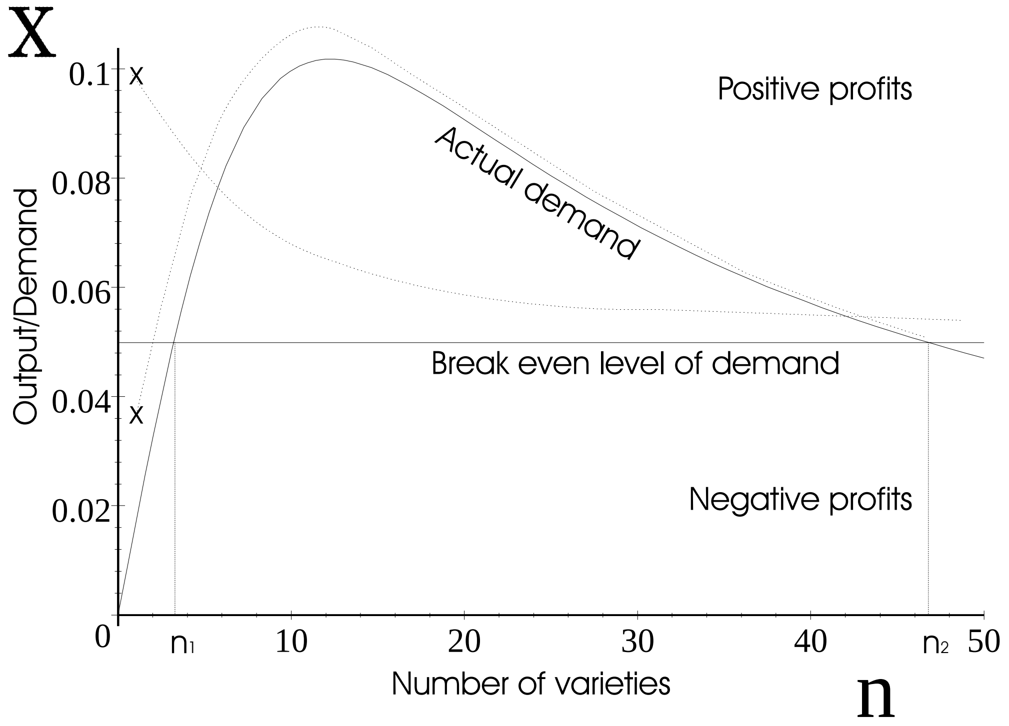

Figure 3: The demand for manufactured goods as a function of the number of varieties. Parameter settings: $L = 750$, $\alpha = 5$, $\beta = 50$, $b_m = 0.1$, $\gamma = 0.5$, $\psi = 0.005$, $b_A = 0.02$

demand for *every* variety. This is rather remarkable. The reason for this is that the elasticity of substitution between sleep and leisure is rather high. With very low numbers of manufactured goods in the market, a high proportion of the peasant's time allocation is spent on sleep. The introduction of new commodities leads to a substitution away from sleep towards leisure and work. The relative "price decrease" implied by the increase in variety, i.e. the relative increase in utility available, is sufficiently large that the consumer rebalances the portfolio of leisure and sleep in such a way that the time devoted to each leisure activity increases. With each additional variety, however, the relative "price reduction" becomes smaller, so that eventually new varieties not only cannibalise the time devoted to sleep, but also take market share away from other varieties already in the market. Another way of thinking about this effect is to note that given percentage changes in the time devoted to sleep initially yield large absolute changes, since with low variety in the market much time is devoted to sleep. With each addition to variety, however, the same percentage changes in sleep time will yield lower and lower absolute changes, so that the increments in income and time available for leisure become progressively lower.

5 Objections

5.1 Monopoly production

A key problem with the account sketched out above is that the monopolistic competition model ceases to perform adequately when the number of varieties is low. One of the key assumptions in deriving the pricing formula 18 is that each manufacturer was a small producer so that he or she could safely ignore the impact that own production would have on overall demand. We noted that the own price effect would be of order $\frac{1}{n}$. Clearly with low n this effect is non-negligible. Indeed it *cannot* be ignored when $n = 1$!

In order to assess whether the analysis presented above remains valid, we consider whether a monopoly producer of a single variety could break even in the economy characterised by the parameters used in the previous section. Monopoly production raises a number of additional concerns. In the first instance, we need to decide how profits or losses from the monopoly firm would be distributed. We ignored this issue above, since we assumed that in equilibrium no profits would be made. The simplest addition to the model would be to assume that all profits or losses are simply credited to the workers of the monopoly firm. This means that market demand is given by

$$\begin{aligned} X_m &= \phi L (1-\gamma) \frac{\pi_m^{r-1} w}{(\pi_m^r + w^r)} + (1-\phi) L (1-\gamma) \frac{\pi_m^{r-1}(w+I)}{(\pi_m^r + w^r)} \\ &= L(1-\gamma) \frac{\pi_m^{r-1}(w + (1-\phi) I)}{(\pi_m^r + w^r)} \end{aligned} \tag{28}$$

where I is the profit/loss per manufacturing worker.

Since I is in turn dependent on the price charged and the quantity sold there is a potentially nasty endogeneity lurking in the firm's decision making problem. We assume that the firm ignores its effect on I and calculates the elasticity of demand taking I as constant. This yields the more tractable expression

$$\begin{aligned} \frac{\partial X}{\partial p}\frac{p}{X} &= L(1-\gamma)(w+(1-\phi)I) \frac{\pi_m^{r-2}(w^r(r-1) - \pi_m^r)}{(\pi_m^r + w^r)^2} \frac{p(\pi_m^r + w^r)}{L(1-\gamma)\pi_m^{r-1}(w+(1-\phi)I)} \\ &= \frac{p}{\pi_m}\left(r \frac{w^r}{\pi_m^r + w^r} - 1 \right) \end{aligned}$$

The mark-up equation is given by

$$\begin{aligned} p &= \frac{1}{1 - \frac{1}{|\varepsilon_i|}} \beta w \\ &= \left(\beta - \frac{(\beta + b_m)}{r} \left(\frac{\pi_m^r + w^r}{w^r} \right) \right) w \end{aligned}$$

which can be written more elegantly as

$$(p - \beta w) = -\left(\frac{\pi_m^r + w^r}{r w^{r-1}} \right)(\beta + b_m) \tag{29}$$

The firm will be able to realise a profit only if $r < 0$, i.e. $0 < \rho < 1$. This condition is met in the case considered in the previous section. Observe that $\pi_m = p + w b_m$, so this equation defines p implicitly in terms of the other parameters. With the settings used in section 4 we find that $p = 20120$. This is 33% lower than the corresponding value obtained through the monopolistic pricing formula in equation 18. Firms that understand their impact on consumer demand will price lower than we supposed in the previous section.

How will this affect the break-even level reflected in Figure 3? While the firm will charge a lower price, it has to meet the same fixed costs of production as before. Consequently the break-even level of demand will be *above* that given in Figure 3. In the case of the monopolist it would need to sell at least 0.09 88 units before the firm made positive profits. We have indicated this point in Figure 3 by means of an "x". The true break-even curve will look something like the dotted decreasing curve in that Figure. The monopolistic competition pricing formula should become progressively more accurate as n increases.

The higher break-even point is offset to some extent, however, by higher demand. Since the monopoly charges lower prices than we had assumed in our model, actual demand will also be higher. We can calculate the actual demand using equation 28 and assuming zero profits. This point is also indicated in Figure 3 by means of an "x". It is located at $X_m = 0.03661\,7$. We have again sketched in the true zero-profit demand schedule given the actual pricing behaviour by means of a dotted curve.

The numbers derived in the case of the monopoly show that the existence of multiple equilibria is not simply due to the bad performance of the monopolistic pricing competition model at low n. It is indeed impossible for a single monopolist to produce profitably in this economy. There will be critical value n_1 (plausibly even larger than 3) at which it becomes possible to produce profitably. In short, the qualitative predictions from the model do not change.

5.2 The elasticity of substitution between sleep and leisure

One assumption that is, however, critical in generating the behaviour discussed above is that the elasticity of substitution between sleep and leisure *as a group* is higher than the elasticity of substitution between different varieties of leisure, i.e. $\rho > \frac{\sigma-1}{\sigma}$. In the example above the elasticity of substitution between sleep and the composite leisure commodity is $\frac{1}{1-\rho} = 2$, while the elasticity of substitution between different varieties of leisure is $\sigma = 1.5$. Indeed, the market demand curve (equation 24) will exhibit the "humped" form shown in Figure 3 if, and only if, $\frac{r}{1-\sigma} > 1$. If $\frac{r}{1-\sigma} \leq 1$ then the curve is monotonically decreasing and there will be at most one point of intersection with the break-even curve.

The key question is therefore whether it is sensible to posit such a high elasticity. At one level this would seem to be an empirical question. At another level, however, this points to the crucial nature of the shape of the market demand curve given in Figure 3. The fact that demand is increasing with additional variety at low n is essential for the generation of multiple equilibria. This pattern can, however, be rationalised on bases other than a high elasticity of substitution. It would follow, for instance, if the manufactured goods acted as *complements* at low n. Indeed it is plausible that they would. Some goods are linked through a particular lifestyle. Colonial adminstrators understood this very well when they tried to persuade subjugated populations to shift from their current way of life to a more "respectable" one. Demand for particular types of clothing , bedding and other accessories was all interlinked in this way. Consequently the conversion from subsistence agriculture to market production tended to happen in groups – some communities adopted "Western" lifestyles and moved to market production, while others did not. Indeed "conversion" (to Christianity) was generally a concomitant process.

If commodities provide additional utility if they are consumed together, then it is easy to see how demand might be deficient at low levels of industrialisation. The introduction of new complementary varieties creates externalities for all varieties currently being consumed, leading to an overall increase in demand which in turn makes it easier for all firms to meet their break-even requirements.

5.3 Closed economy

The model essentially assumes a closed economy. In that it is not any different from similar models, such as the "big push" model (Murphy et al. 1989b). The justification for this assumption is that in real economies there are generally enough frictions that local demand continues to be an important determinant of the level at which firms can produce. Low local demand can therefore prevent firms from reaching the economies of scale which might enable them to compete effectively internationally.

In our particular model, however, it is clear that trade might affect the demand side in interesting ways also. If the required portfolio of consumption items could all be imported, then the labour supply effect documented above might start even in the absence of domestic production. Of course the effect might be stronger if local producers could produce the commodities cheaper. On the other hand, with increasing returns to scale local producers might never be able to enter the market.

One context in which the closed economy assumption is unambiguously warranted is in the very first process of industrialisation, i.e. the change from feudalism to market production in Britain. The model suggests that it might be very difficult to get industrialisation started if it requires the simultaneous availability of a number of complementary consumer items. An argument along these lines (from a different perspective) is advanced by Brenner (1986). He suggests that in the absence of consumer markets the rational response for a peasant is to try to meet all essential needs through home production, i.e. to *diversify* own production rather than to specialise. In this context specialised production (which is a form of increasing returns to scale) can never get started, since there is always deficient demand for its output. The problem is that each producer has to be certain that the market for the complementary products and essential necessities will all function simultaneously.

6 Industrialisation through coerced labour

We turn now to consider state intervention in this context. From the point of view of potential industrialists the "development problem" is experienced both as deficient demand (for the output) and deficient supply (of labour). The deficient demand might be overcome through a coordinated investment strategy of the "big push" variety. Note, however, that this does not solve the labour supply issue. Similar development traps have been experienced in a number of contexts. One of the "solutions" that the British colonial state devised was to coerce peasant producers into the modern economy. One way of doing so was to levy poll taxes which had to be paid in cash. The only way of raising the cash was for members of the peasant households to sell their labour on the market to modern firms or to sell some of their agricultural output.

We can sketch out the effects of such an intervention in our model. We assume that the government levies a flat rate poll tax of τ. This means that total income of every agent is $wT^* - \tau + I$, where we have allowed the possibility that some agents may have income from profits I. The individual demand equations 6, 8a and 8b continue to hold (suitably modified), as does the general formulation of the labour supply equation (equation 10). It is immediately clear that consumption of the agricultural good declines. Consumption of the manufactured commodity will be lower than it would have been in the situation where there was no tax, but that is not the relevant comparison, since there was no industry in that state. The labour supply equation shows an unambiguous increase in the quantity of labour supplied.

The aggregate tax revenue is $L\tau$. We assume that this is handed over as a lump sum subsidy to the single monopoly producer. The market demand for this producer will now be given by

$$X_m = L\left(1-\gamma\right)\frac{\pi_m^{r-1}\left(w-\tau+\left(1-\phi\right)I\right)}{\left(\pi_m^r+w^r\right)} \tag{30}$$

(compare with equation 28). The monopoly firm's pricing decision, however, turns out to be precisely as before, i.e. the price is given implicitly by equation 29.

The total profits made by this firm will be given by

$$\begin{aligned}\left(1-\phi\right)LI &= pX_m - wL_mT_{wm} + L\tau \\ &= \left(p-\beta w\right)X_m - \alpha w + L\tau \\ &= -\left(\frac{\pi_m^r+w^r}{rw^{r-1}}\right)\left(\beta+b_m\right)X_m - \alpha w + L\tau \end{aligned} \tag{31}$$

This equation implicitly fixes the quantity $(1-\phi)I$ in terms of τ. There is no reason to expect *a priori* that these *per capita* profits should be zero. Furthermore two other markets have to clear: the supply of industrial workers has to be such that the level of industrial production matches the level of demand; and these industrial workers have to be fed!

Analysing balance in the industrial sector first, the level of demand given in equation 30 has to be consistent with the level of production, given by equation 16. Given the fact that the full income of every industrial worker is $w-\tau+I$, the work effort of these workers will be given by

$$T_{wm} = \gamma\left(\frac{1+b_A\left(\tau-I\right)}{1+wb_A}\right)+\left(1-\gamma\right)\left(\frac{\pi_m^{r-1}p+\left(\tau-I\right)\left(b_m\pi_m^{r-1}+w^{r-1}\right)}{\pi_m^r+w^r}\right)$$

Substituting this into the production function, we find that the labour market for industrial workers will balance if

$$\begin{aligned}&\left(1-\phi\right)L\left[\gamma\left(\frac{1+b_A\left(\tau-I\right)}{1+wb_A}\right)+\left(1-\gamma\right)\left(\frac{\pi_m^{r-1}p+\left(\tau-I\right)\left(b_m\pi_m^{r-1}+w^{r-1}\right)}{\pi_m^r+w^r}\right)\right] \\ =\;& \alpha+\beta L\left(1-\gamma\right)\frac{\pi_m^{r-1}\left(w-\tau+\left(1-\phi\right)I\right)}{\left(\pi_m^r+w^r\right)}\end{aligned} \tag{32}$$

Demand for the agricultural good is given by

$$X_A = L\gamma\frac{w-\tau+\left(1-\phi\right)I}{1+wb_A}$$

Substituting this expression into the agricultural production function 14 and noting that

$$T_{wA} = \gamma\left(\frac{1+b_A\tau}{1+wb_A}\right)+\left(1-\gamma\right)\left(\frac{\pi_m^{r-1}p+\tau\left(b_m\pi_m^{r-1}+w^{r-1}\right)}{\pi_m^r+w^r}\right)$$

Parameter	Peasant	Monopoly with subsidy	
		Peasant	Proletarian
ϕ	1	.524	
τ	0	18.987	
I	0	38.153	
full income	200	181.01	219.17
Z_A	20.0	18.101	21.917
Z_m	0	4.4187×10^{-5}	5.3501×10^{-5}
T_S	.5	.44808	.54253
T_w	.1	.18989	0.019135
U	3.1623	2.8762	3.1649
Aggregate Welfare	2371.7	2260.1	

Parameter settings: $L = 750$, $\alpha = 5$, $\beta = 50$, $b_m = 0.1$, $\gamma = 0.5$, $\psi = 0.005$, $b_A = 0.02$

Table 2: Monopoly industry supported by poll tax vs Peasant Equilibrium

it follows that our equilibrium condition can be written as

$$\phi\left[\gamma\left(\frac{1+b_A\tau}{1+wb_A}\right)+(1-\gamma)\left(\frac{\pi_m^{r-1}p+\tau\left(b_m\pi_m^{r-1}+w^{r-1}\right)}{\pi_m^r+w^r}\right)\right]=\psi\gamma\frac{w-\tau+(1-\phi)I}{1+wb_A} \tag{33}$$

Equations 31, 32 and 33 are three equations in the three unknowns τ, ϕ and I. If τ is set so as to clear both markets, then it is evident that this will, in general, be at a level where I is non-zero. In fact it seems clear that I must be positive. If it were negative then there would be no incentive for any agricultural worker to leave the rural areas and migrate to the cities. Consider the situation depicted in Figure 3 again. The fact that the monopoly does not break even without assistance (the positions indicated by the cross) is reflected in two markets not clearing – demand for the industrial product is insufficient, but the supply of labour will be deficient too. If we were to make a transfer to the monopoly sufficient to cover the shortfall, but funded this transfer exclusively through external funding (e.g. transfers from abroad), then the monopoly would still be hamstrung by the shortage of labour. The poll-tax must be set at a level higher than the *per capita* gap between actual demand and the break-even level of demand shown in Figure 3 for two reasons: firstly the introduction of the tax lowers incomes and hence drops the demand curve shown in that figure. Secondly the tax has to be high enough to ensure the necessary labour supply response.

If I is positive it follows that agricultural workers work longer hours and sleep less than industrial workers. Industrialisation through the introduction of a poll tax is therefore likely to also introduce inequaltiy. It is unclear *ex ante* what such a policy would do to aggregate welfare. Income of the agricultural workers is lower than in the "peasant economy", but the consumption of the additional product might increase utility so that the coercive equilibrium might actually Pareto dominate the peasant equilibrium. This seems highly unlikely however.

With the parameter settings identical to those considered in the previous sections the outcome is as indicated in Table 2. Aggregate welfare was calculated on strictly utilitarian principles, i.e. it is simply the sum of the individual utilities. In this case it is clear that aggregate utility is lower after the forced episode of industrialisation than it was in the peasant equilibrium. Nevertheless the impact is differential: the urban workforce is better off while the agricultural producers are much worse off. This of course cannot really be an equilibrium either – we would now expect to see rural to urban migration up to the point where real incomes (after tax and subsidies) are equal. This would lead to disequilibria in the factor and output markets and unemployment of the Harris-Todaro type (Harris and Todaro 1970).

Neverteless it is clear a government intervention of the poll tax type could be effective in establishing an industrial sector – even if this is quite a distorted sector initially. By increasing the tax rate yet further, it could subsidise the establishment of the second and then the third firm. In our model industrialisation would be self-sustaining once the critical threshold of $n = 3.2$ had been exceeded (see Table 1). This would allow a reduction of the tax rates and a rebalancing of the urban-rural relationship. Nevertheless it is not clear what the political economy ramifications of this particular "development path" might be. The historical record suggests that attempts to subsidise industrial development through heavy taxation of the rural areas (such as in the Soviet Union) inevitably create resistance which leads to a reduction of overall output. Furthermore it is not clear that once the urban sector has become used to being subsidised that it would take kindly to having this benefit removed.

7 Conclusion

The model we have presented shows that there may very well be a connection between industrialisation and surplus labour of the type suggested by the early development economists. It suggests, however, that the transition process from a completely agrarian (feudal) society to a society based on market exchange may be more complicated than they envisaged. Indeed our analysis suggests why forcible transfers from rural populations to the urban sector seem to have occurred in a number of contexts. Whether our analysis is relevant to "real world" processes is, of course, the key question.

Our model depends on the following relationships:

- There must be fairly elastic substitutability between sleep/leisure and other uses of time (production, consumption)
- There must be significant fixed costs/increasing returns to scale in industrial production
- There must be complementarities in the consumption of industrial goods, particularly at the initial stages.

There are a number of studies that suggest that at least some of the mechanisms posited in this paper exist. Biddle and Hamermesh (1990), for instance, show empirically that sleep time responds in predictable ways to economic incentives. Szalontai (2004) has recently shown that the same mechanisms operate in a developing country context, viz. South Africa. Indeed he showed that the difference in **mean** sleep time between African women (who are particularly poor) and White men (who are much more affluent) is of the order of ninety minutes per day. All of this difference can be explained in terms of the relevant economic variables.

Gronau and Hamermesh (2001) have used time-use data to show that variety seems to be prized. This finding has been confirmed in the South African context by Manga (2004). We have used demand for variety as a mechanism for modelling complementarities in consumption.

The idea that there are likely to be increasing returns to scale in industrial production has become much more common place in the literature on development. Indeed the citation from Krugman (1995), reproduced earlier, suggested that understanding the likely impact of economies of scale should be at the core of development economics. Our analysis reconnects the topic of surplus labour to that concern. Indeed it suggests that understanding the relationship between agriculture (the constant returns to scale sector) and industry in the process of development is likely to be crucial.

While our model suggests that there might be "excess" labour in the rural areas in the sense that given the right incentives much of this labour could be transferred to the urban areas without loss of production, it is not "surplus labour" in the sense that Georgescu-Roegen (1960) would have recognised it. Indeed it is not a situation in which the marginal product of labour is zero. Even in the "peasant economy" aggregate output could be increased if the peasants slept less. Nevertheless the amount of leisure consumed by these peasants does not explain why they are poor – they are poor because the high level industrial equilibrium is not available. Making the jump to this new equilibrium requires simultaneously the creation of output markets and urban labour markets. Sacrificing leisure now reduces immediate utility but does not create the preconditions for the industrial equilibrium.

The model has some other interesting ramifications. Comparing the two equilibria given in Table 1 we observe that both societies have the same wage rate. This means that "full income" in both societies is also identical. Consumption of the agricultural commodity is identical. Prices are identical. The only difference is that in the one situation people consume more leisure goods, work more and sleep less, while in the other they have fewer choices available. Well-being is definitely higher in the high variety economy. This suggests that traditional measures of well-being might not be enough – we may very well need to take into consideration factors such as how varied people's lives are. Analyses of time use (see Hamermesh and Pfann, eds 2005) may therefore be very useful adjuncts to more traditional economic analyses. Indeed our model suggests that time allocation issues may be central in development.

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